**Secondary Mathematics Scheme of Work: Stage 11**

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| *Unit* | *Lessons* | *Key ‘Build a Mathematician’ (BAM) Indicators* | *Essential knowledge* |
| [Investigating properties of shapes](#IPS) | 16 | * [Simplify surds, including rationalising the denominator of a surd expression](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M1_BAM.pdf)
* [Manipulate quadratic expressions by completing the square](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M2_BAM.pdf)
* [Deduce roots and turning points of quadratic functions](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M3_BAM.pdf)
* [Understand the concept of an instantaneous rate of change](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M4_BAM.pdf)
* [Sketch translations and reflections of given functions](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M5_BAM.pdf)
* [Solve quadratic inequalities in one variable](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M6_BAM.pdf)
* [Use the sine and cosine rules to solve problems](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M7_BAM.pdf)
 | * Know that $\sqrt{a\pm b} $ $\ne $ $\sqrt{a }\pm \sqrt{b }$, $\sqrt{\frac{a}{b}}= \frac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{a × b} $ $=$ $\sqrt{a }× \sqrt{b }$
* Know the formula for solving quadratic equations
* Know function notation
* Know graphs of exponential and trigonometric functions
* [Know the sine rule, a/sinA = b/sinB = c/sinC](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* [Know the cosine rule, a² = b² + c² - 2bc cosA](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* [Know area of triangle = ½ab sinC](http://kangaroomaths.com/free_resources/display/trigonometry.pdf)
* Know that histograms should be plotted using frequency density when groups are of unequal widths
 |
| [Calculating](#CALC) | 6 |
| [Solving equations and inequalities I](#SEI1) | 12 |
| [Mathematical movement I](#MM1) | 3 |
| [Algebraic proficiency: tinkering](#APT) | 5 |
| [Proportional reasoning](#PR) | 5 |
| [Pattern sniffing](#PS) | 4 |
| [Solving equations and inequalities II](#SEI2) | 6 |
| [Algebraic proficiency: visualising I](#APV1) | 7 |
| [Analysing statistics](#AS) | 5 |
| [Algebraic proficiency: visualising II](#APV2) | 3 |
| [Mathematical movement II](#MM2) | 4 |
| Total: | 76 |  |  |

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| **Maths Calendar** | *Based on 8 maths lessons per fortnight* |
| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 |
| [Investigating properties of shapes](#IPS) | [Calculating](#CALC) | [Solving equations and inequalities I](#SEI1) | [Movement I](#MM1) | [Tinkering](#APT) | [Prop'l reasoning](#PR) | [Patterns](#PS) |  |
| [11M7 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M7_BAM.pdf) | [11M1 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M1_BAM.pdf) | [11M2 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M2_BAM.pdf), [11M3 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M3_BAM.pdf) |  |  |  |  |  |
| Week 14 | Week 15 | Week 16 | Week 17 | Week 18 | Week 19 | Week 20 | Week 21 | Week 22 | Week 23 | Week 24 | Week 25 | Week 26 |
| Assessment | [Solving equations II](#SEI2) | [Visualising I](#APV1) | [Analysing statistics](#AS) | [Visualising II](#APV2) | [Movement II](#MM2) | The Final Countdown |
|  | [11M6 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M6_BAM.pdf) | [11M5 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M5_BAM.pdf) |  | [11M4 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M4_BAM.pdf) |  |  |
| Week 27 | Week 28 | Week 29 | Week 30 | Week 31 | Week 32 | Week 33 | Week 34 | Week 35 | Week 36 | Week 37 | Week 38 | Week 39 |
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| *Investigating properties of shapes* | *16 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * know the formulae for Pythagoras’ theorem, a² + b² = c², and apply it to find lengths in three dimensional figures
* know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent and apply them to find angles and lengths in three dimensional figures
* know and apply the sine rule, a/sinA = b/sinB = c/sinC, and the cosine rule, a² = b² + c² - 2bc cosA, to find unknown lengths and angles
* know and apply area = ½ab sinC to calculate the area, sides or angles of any triangle
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore three-dimensional shapes
* Apply Pythagoras’ theorem in three dimensions
* Apply trigonometry in three dimensions
* Know and use the sine rule
* Know and use the cosine rule

**Bring on the Maths: GCSE Higher Shape**Investigating triangles: #6Investigating angles: #10, #13 | * Use Pythagoras’ theorem to find the length of a given diagonal in a cuboid
* Use Pythagoras’ theorem to find any length in a cuboid
* Use Pythagoras’ theorem to find missing lengths in other three dimensional figures
* Use Pythagoras’ theorem to solve problems involving three dimensional figures
* Use trigonometry to find the angle between a line and a plane
* Solve simple problems involving missing lengths and angles in three dimensional figures
* Solve more complex problems involving missing lengths and angles in three dimensional figures
* Know and use the sine rule in simple cases
* Use the sine rule to find a missing side in a non-right angled triangle
* Use the sine rule to find a missing angle(s) in a non-right angled triangle
* Know and use the cosine rule in simple cases
* Use the cosine rule to find a missing side in a non-right angled triangle
* Use the cosine rule to find a missing angle in a non-right angled triangle
* Solve complex problems involving bearings
* Know and use area = ½ab sinC to calculate the area of any triangle
* Know and use area = ½ab sinC to calculate sides or angles of any triangle
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Apply Pythagoras’ theorem in two dimensions
* Know the trigonometric ratios, sinθ = opp/hyp, cosθ = adj/hyp, tanθ = opp/adj
* Choose an appropriate trigonometric ratio that can be used in a given two-dimensional situation
* Set up and solve a trigonometric equation to find a missing side or angle in a right-angled triangle
 | Diagonal (Face Diagonal, Space Diagonal)PlaneOpposite, Adjacent, HypotenuseTrigonometrySine, Cosine, TangentAngle of elevation, angle of depression**Notation**sinθ stands for the ‘sine of θ’sin-1 is the inverse sine function, and not 1 ÷ sin | Ensure that all students are aware of the importance of their scientific calculator being in degrees mode.Ensure that students do not round until the end of a multi-step calculationThis unit of trigonometry should focus on right-angled triangles in three dimensions and non-right-angled triangles. NRICH: [History of Trigonometry](http://nrich.maths.org/6843)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students explore how to derive the sine rule* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a diagonal/right angle in this cuboid. And another, and another …
* Convince me that you have chosen the correct trigonometric fact
* What’s the same and what is different: a² = b² + c² - 2bc cosA and a² = b² + c². Can you find a connection?
* What’s the same and what is different: area = ½ab sinC and area = ½bh. Can you find a connection?
 | KM: Investigate Euler bricksHwb: [Q8 Triangle Side Length](http://hwb.wales.gov.uk/Resources#resource/3eae7de1-37a0-4a40-9707-e60a20a40c92/en)Hwb: [T3 Greenhouse](http://hwb.wales.gov.uk/Resources#resource/51080f3f-4ee0-4607-9479-94a86226bf9c/en)Hwb: [T20 Wardrobe](http://hwb.wales.gov.uk/Resources#resource/e1a3cc41-2a9d-468f-ad8b-611d3c530518/en)NRICH: [Raising the Roof](http://nrich.maths.org/5614)NRICH: [Coke Machine](http://nrich.maths.org/265)NRICH: [Cosines Rule](http://nrich.maths.org/351)**Learning review**KM: [11M7 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M7_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may label opposite and adjacent in a non-right-angled triangle
* Some students may not balance an equation such as 5 = 4/sinθ correctly, believing that the next step is sinθ = 5/4
* Some students may think that cos-1θ = 1 ÷ cosθ
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| *Calculating* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3) and rationalise denominators
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Manipulate expressions by simplifying surds

**Bring on the Maths: GCSE Higher Number**Investigating numbers: #4, #5 | * Know and use $\sqrt{a × b} $ $=$ $\sqrt{a }× \sqrt{b }$
* Simplify surds
* Solve problems involving the simplification of surds
* Multiply two binomials involving surds
* Rationalise the denominator of a simple surd expression
* Rationalise the denominator of a more complex surd expression
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Calculate exactly with surds
* Use the functionality of a scientific calculator when calculating with roots and powers
 | Power, RootIndex, IndicesSurdSimplifyRationalise**Notation**$\sqrt{a }$ represents the ‘positive square root of’, and the bar should be used to enclose contents correctly | Surd is derived from the Latin ‘surdus’ (‘deaf’ or ‘mute’). A surd is therefore a number that cannot be expressed (‘spoken’) as a rational number. Students should already have established the following facts: $\sqrt{a\pm b} $ $\ne $ $\sqrt{a }\pm \sqrt{b }$, $\sqrt{\frac{a}{b}}= \frac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{a × b} $ $=$ $\sqrt{a }× \sqrt{b }$ NCETM: [Departmental workshops: Surds](https://www.ncetm.org.uk/resources/13234)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All students carry out the Standard Unit activity referenced below* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always/Sometimes/Never: tan 45° = $\frac{1}{\sqrt{2}}$
* What’s the same and what is different: $\left(\sqrt{3 }+ \sqrt{5 }\right)^{2}$ and $\left(\sqrt{3 }+ \sqrt{5 }\right)\left(\sqrt{3 }- \sqrt{5 }\right)$ ?
* Show me an expression, of the form $a\sqrt{b }$, that is equivalent to $24\sqrt{3 }$. And another, and another …
 | Hwb: [Q3 Manipulating surds](http://hwb.wales.gov.uk/Resources#resource/b65d1455-f3a6-47ac-8378-fb33e7171d31/en)Standards Unit: [N11 Manipulating surds](http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Number%20lessons/N11%20-%20Manipulating%20Surds.pdf)NRICH: [Surds](http://nrich.maths.org/620)**Learning review**KM: [11M1 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M1_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may think that $\sqrt{a\pm b} $= $\sqrt{a }\pm \sqrt{b }$
* Some students may think that $\left(\sqrt{a }+ \sqrt{b }\right)^{2}=a+b$
* Some students may write √4 × 3 when they should write $\sqrt{4×3}$ (or √(4 × 3))
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| *Solving equations and inequalities I* | *12 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve quadratic equations by completing the square and by using the quadratic formula
* deduce turning points of quadratic functions by completing the square
* deduce roots of quadratic functions algebraically
* work with general iterative processes
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve quadratic equations
* Solve practical problems involving quadratic equations
* Understand and use iterative processes

**Bring on the Maths: GCSE Higher Algebra**Solving quadratic equations: #6, #8, #9, #10, #11, #12 | * Complete the square for a quadratic expression (a = 1)
* Complete the square for a quadratic expression (a > 1)
* Solve a quadratic equation (a = 1) by completing the square
* Solve a quadratic equation (a > 1) by completing the square
* Deduce the turning point of a quadratic function by completing the square
* Deduce the roots of a quadratic function using the completed square form
* Know and apply the formula for solving a simple quadratic equation of the form *ax*² + *bx* + *c* = 0
* Know and apply the formula for solving more complex quadratic equation of the form *ax*² + *bx* + *c* = 0
* Solve equations involving fractions that can be rearranged into the form ax² + bx + c = 0
* Solve problems in probability that generate a quadratic equation
* Solve problems involving quadratic equations
* Derive an iterative formula that can be used to find approximate solutions to a complex equation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Solve a quadratic equation by rearranging and factorising
* Identify when a quadratic equation cannot be solved by factorising
* Calculate fluently with negative numbers
* Rearrange algebraic expressions and equations
* Understand and use interval bisection
* Rearrange an equation to form an iterative formula
 | (Quadratic) equationFactoriseRearrangeComplete the squareUnknownManipulateMaximum, minimumParabolaRecurrence relationInterval bisection**Notation**The form (*x* + *p*)2 – *q* usually implies that completing the square is requiredRecurrence relations are equations such as *xn*+1 = 2*xn* - 3 | *Problems involving quadratic equations include, for example, shapes with dimensions expressed algebraically and area given.**Students have previously explored a range of iterative processes. They should now choose an appropriate method given the situation they are faced with.***Common approaches***All pupils investigate geometric representations of completing the square**All pupils derive the formula for solving a quadratic equation by completing the square* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always / sometimes / never: a quadratic equation has two solutions (justify using values of *a*, *b* and *c*)
* Show me an example of a quadratic equation with one solution. And another, and another, …
* Explore geometric representations of completing the square. Make connections between the geometry and the algebra to make sense of the name of the process
 | NRICH: [Proof sorter – quadratic equation](http://nrich.maths.org/1394)NRICH: [Geometric parabola](http://nrich.maths.org/6965)**Learning review**KM: [11M2 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M2_BAM.pdf), [11M3 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M3_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some students may attempt to always substitute positive values for *a*, *b* and *c* when using *ax*² + *bx* + *c* = 0
* Some students may forget that squaring a negative number results in a positive solution
* Some students may think that (*x* + *p*)2 – *q* implies that *p* must be positive
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| *Mathematical movement I* | *3 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including negative scale factors)
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore enlargement of 2D shapes

**Bring on the Maths: GCSE Higher Shape**Manipulating shapes I: #5 | * Use the centre and scale factor to carry out an enlargement of a 2D shape with a negative scale factor
* Find the scale factor of an enlargement with negative scale factor
* Find the centre of an enlargement with negative scale factor
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive scale factor
 | Scale FactorSimilarTransformationEnlargement | Pupils have identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9. Stage 10 included enlargement using a fractional scale factor.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using negative scale factors* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always/ Sometimes/ Never: The resulting image of an enlargement is larger than the original object
 | KM: [Enlargement 3](http://kangaroomaths.com/free_resources/teaching/geometry/enlargement_3.docx)NRICH: [Transformation game](https://nrich.maths.org/5457)**Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that the resulting image of an enlargement has to be larger than the original object.
* Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship.
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| *Algebraic proficiency: tinkering* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * interpret the succession of two functions as a ‘composite function’
* interpret the reverse process as the ‘inverse function’
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve problems involving functions
 | * Understand the meaning of a function
* Know and use the notation for composite functions
* Solve problems involving composite functions
* Find the inverse of a given function
* Solve problems involving inverse functions
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Given a function, establish outputs from given inputs
* Given a function, establish inputs from given outputs
* Use a mapping diagram (function machine) to represent a function
* Use an expression to represent a function
 | MappingFunctionInverse functionComposite function**Notation***f*(*x*) for a function of *x**f*-1(*x*) for the inverse of a function, *f*(*x*)*fg*(*x*) for a function (*f*) of a function (*g*) of *x* | In Stage 7, pupils have learnt about interpreting simple expressions as functions with inputs and outputs including the use of mapping diagrams. They will not have met formal function notation such as *f*(*x*), *f*-1 (*x*) and *fg*(*x*) until this unit.Some pupils may think that *fg*(*x*) means ‘do *f*(x) first then *g*(*x*)’ rather than the function *f*(*x*) operating on the output of the function *g*(*x*).The graph of the inverse function is the reflection of the graph of the function reflected in the line *y* = *x*.Note that OCR do not require students to have knowledge of function notation.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)NCETM: [Secondary Magazine October 2016 ‘It Stands to Reason’](https://www.ncetm.org.uk/resources/49564)**Common approaches***f(g(x) is interpreted as the function f(x) operating on the output of the function g(x)* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me function and the corresponding inverse function. And another, and another, …
* Convince Kenny *g*(*x*) = $\frac{x-3}{2}$ is the inverse function of *f*(*x*) = 2*x* + 3
* Always/Sometimes/Never: *fg*(*x*) = *gf*(*x*)
* Find a function whose inverse is the same function
 | KM: [Functions introduction](http://kangaroomaths.com/free_resources/teaching/algebra/functions_introduction.pptx)**Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think *f*-1(*x*)= $\frac{1}{f(x)}$
* Some pupils may think that *fg*(*x*) means ‘do *f*(*x*) first then g(*x*)’
* Some pupils may think that *ff*(x) means (*f*(*x*))2
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| *Proportional reasoning* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Ratio and Proportion progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_RatioProportion.xlsx) |
| * construct equations that describe direct and inverse proportion
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore differences between direct and inverse proportion
* Solve problems involving proportion

**Bring on the Maths: GCSE Higher Number**Ratio and Proportion: #4, #5 | * Construct and use simple equations describing direct proportion
* Construct and use more complex equations describing direct proportion
* Construct and use simple equations describing inverse proportion
* Construct and use more complex equations describing inverse proportion
* Solve problems involving direct and inverse proportion
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recognise a graph that illustrates direct or inverse proportion
* Interpret equations that describe direct or inverse proportion
* Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
* Solve problems which include finding the multiplier in a situation involving direct or inverse proportion
 | Direct proportionInverse proportionMultiplier**Notation**$∝$ - ‘proportional to’ | In Stage 9, pupils have learnt about solving problems involving direct and inverse proportion, including graphical and algebraic representations. In Stage 10, they learnt about interpreting equations that describe direct and inverse proportion.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)NCETM: [Department Workshop – Proportional Reasoning](https://www.ncetm.org.uk/resources/10334) **Common approaches***α is read as “proportional to”**k is used as the ‘constant of proportionality’ – i.e. if y α x then y = kx* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me two pairs of quantities that are directly proportional. And another pair. And another pair…
* Show me two pairs of quantities that are inversely proportional. And another, and another, …
* Convince Kenny that ‘X is inversely proportional to Y’ is equivalent to ‘X is proportional to 1/Y’
 | NRICH: [Triathlon and Fitness](http://nrich.maths.org/7586)OCR Maths: [Lesson Element – Inverse Proportion](http://www.ocr.org.uk/Images/topic-5.02b-inverse-proportion-teacher-instructions-lesson-element.pdf)AQA Maths: [Ratio, Proportion and Change](http://allaboutmaths.aqa.org.uk/1030) **Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that *y* is inversely proportional to *x* means y = $\frac{x}{k}$
* Some pupils may interpret inverse proportion relationships as direct proportion
* Some pupils may think that the proportionality constant has to be greater than 1.
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| *Pattern sniffing* | *4 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 or a surd) and other sequences
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * investigate geometric progressions
 | * Recognise and use geometric progressions, ar^n, when r is a fraction > 0
* Recognise and use geometric progressions, ar^n, when r is a surd
* Solve problems involving geometric sequences
* Recognise and use non-standard sequences
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the difference between an arithmetic progression, a quadratic sequence and a geometric progression
* Recognise a simple geometric progression
* Find the next three terms in a geometric progression
* Find a given term in a simple geometric progression
* Describe a geometric progression
 | Termnth termFirst (second) differenceGeometric ProgressionSurd**Notation**T(n) is often used to indicate the ‘nth term’rn | In Stage 10, pupils have learnt about recognising and using simple geometric progressions where n is an integer, and r is a rational number > 0. In this unit, the common ratio (r) could be a **surd**. Pupils are also introduced to non-standard sequences such as $\frac{1}{1×2}, \frac{1}{2×3}, \frac{1}{3×4}, \frac{1}{4×5}, …$NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are introduced to geometric sequences with r as a rational number using the ‘*[*Kangaroo Problem*](http://www.kangaroomaths.com/free_resources/ks5/resources/session4/gs/kangaroolovers.doc)*‘* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a geometric progression. And another, and another, …
* Convince Kenny that 1, 0.5, 0.25, 0.125 is a geometric sequence.
* Convince Jenny that 3, 3$\sqrt{2}$, 6, 6$\sqrt{2}$, 12, … is a geometric sequence.
* Always/Sometimes/Never: The ratio (r) of terms in a geometric sequence is greater than 1
 | KM: [Kangaroo Problem](http://www.kangaroomaths.com/free_resources/ks5/resources/session4/gs/kangaroolovers.doc)NRICH: [Summing Geometric Progressions](http://nrich.maths.org/8054)AQA Maths: [Sequences](http://allaboutmaths.aqa.org.uk/index.php?CurrMenu=1022)Resourceaholic: [Sequences](http://www.resourceaholic.com/2015/07/sequences.html)**Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may have difficulty with sequences when r is a surd
* Some pupils think the ratio (r) of terms in a geometric sequence has to be greater than 1
* Some pupils may have difficulty spotting the position-to-term relationship for ‘non-standard’ sequences such as: $\frac{1}{1×2}, \frac{1}{2×3}, \frac{1}{3×4}, \frac{1}{4×5}, …$
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| *Solving equations and inequalities II* | *6 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * solve quadratic inequalities in one variable
* solve two simultaneous equations in two variables where one is quadratic algebraically
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve inequalities
* Solve simultaneous equations

**Bring on the Maths: GCSE Higher Algebra**Solving Simultaneous Equations: #4 | * Solve a quadratic inequality (a = 1)
* Solve a quadratic inequality (a > 1)
* Solve simultaneous equations in two variables where one is a simple quadratic equation using substitution
* Solve simultaneous equations in two variables where one is a more complex quadratic equation using substitution
* Make connections between simultaneous equations and graphs
* Solve problems involving linear and quadratic simultaneous equations
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use set notation to list a set of integers
* Use a formal method to solve a linear inequality
* Show a range of values that solve an inequality on a number line
* Sketch a graph of a quadratic functions
* Find the roots of a quadratic function
* Solve two linear simultaneous equations in two variables by substitution
* Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
 | Unknown(Quadratic) inequalityVariableManipulateSolveSolution setSimultaneous equationsSubstitutionElimination **Notation**The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to) | In Stage 9, pupils have learnt about solving linear inequalities in one variable and representing the solution on a number line. In Stage 10, they learnt about solving inequalities in two variables and representing the solution set using set notation and on a graph.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils should sketch the quadratic function to identify the solution set for a quadratic inequality**All pupils should experience plotting graphs of these situations using graph-plotting software* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a quadratic inequality. And another, and another, …
* Kenny thinks the solution to x2 > 9 is x > 3. Jenny thinks the solution to x2 > 9 is x < -3. Who do you agree with? Explain your answer.
* Always/Sometimes/Never: A pair of simultaneous equations in two variables where one is quadratic algebraically will have two solutions
 | AQA Maths: [Inequalities](http://allaboutmaths.aqa.org.uk/index.php?CurrMenu=1012)Resourceaholic: [Inequalities](http://www.resourceaholic.com/2015/05/new-gcse-inequalities.html)**Learning review**KM: [11M6 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M6_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think the solution to x2 > 16 is x > 4.
* Some pupils may express the solution to a quadratic inequality using incorrect notation, e.g. -2 > x < 2, -2 > x > 2
* Some pupils may think a pair of simultaneous equations in two variables where one is quadratic will only one solution
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| *Algebraic proficiency: visualising I* | *7 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * recognise, sketch and interpret graphs of exponential functions y = k^x for positive values of k, and the trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size
* sketch translations and reflections of a given function
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore graphs of exponential functions
* Explore graphs of trigonometric functions
* Investigate the connections between graphs of functions and their translations

**Bring on the Maths: GCSE Higher Algebra**Investigating Graphs I: #9Investigating Graphs II: #6 | * Plot and use the key features of the graph of an exponential function, y = k^x, for positive values of k
* Plot and use the key features of the graph of the trigonometric function y = sin x
* Plot and use the key features of the graph of the trigonometric function y = cos x
* Plot and use the key features of the graph of the trigonometric function y = tan x
* Know the effects of transforming the graph *y* = *f*(*x*):  *f*(*ax*), *af*(*x*), *f*(*x*)+ *a, f*(*x* + *a*), *y* = *f*(-*x*) and *y* = -*f*(*x*)
* Solve simple problems involving the transformation of graphs
* Solve more complex problems involving the transformation of graphs
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recognise, plot and interpret exponential graphs
* Plot graphs of linear, quadratic, cubic and reciprocal functions
* Find sines, cosines and tangents of given angles
 | Exponential Function, equationLinear, non-linearQuadratic, cubic, reciprocal, exponentialParabolaAsymptoteMaximum, minimum, periodGradient, y-intercept, x-intercept, rootSketch, plotArguments**Notation***y* = *mx* + *c**f*(*x*), *f*(*ax*), *af*(*x*), *f*(*x*)+ *a, f*(*x* + *a*) | The use of dynamic geometry is essential for this unit, such as these [Geogebra files](https://www.geogebra.org/material/show/id/8028) to generate the sine, cosine and tangent graphs from the unit circle and these [Autograph Activities](http://www.autograph-maths.com/challenges/) to explore transformations of graphs.Note the graph of y = x2 is useful to explore the impact of most of the transformations but not f(-x). Some pupils find the transformation y = f(x + a) difficult to understand as they think it should be a translation $\left(\begin{matrix}a\\0\end{matrix}\right)$ Some students may ask what happens with, for example, y = (-2)x. This interesting question can be explored [here](https://www.quora.com/Why-doesnt-my-calculator-graph-y-2-x) and [here](http://phantomgraphs.weebly.com/).NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All teachers explain the term ‘exponent’ to help students understand why ‘exponential’ functions are called ‘exponential’**All pupils should experience using dynamic software (e.g. Autograph) to explore graphs of exponential functions y = k^x for positive values of k**All pupils should experience using dynamic software (e.g. Autograph) to explore graphs of trigonometric functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Draw the graph of y = 1x. Convince me it is an exponential function
* What’s the same, what’s the different: the graphs of y = sin x, y = cos x and y = tan x?
* Show me the graph of an exponential function. And another, and another, …
* Convince Kenny that the graph of f(x – 2) is a translation of the graph f(x) by $\left(\begin{matrix}2\\0\end{matrix}\right)$
* Always/Sometimes/Never: The graph of an exponential function, y = kx for positive values of k, does not intersect with the x-axis
 | NRICH: [What’s that graph?](http://nrich.maths.org/7502)AQA Maths: [Transforming Graphs](http://allaboutmaths.aqa.org.uk/1028)AQA Maths: [Further Sketching Graphs](http://allaboutmaths.aqa.org.uk/1010)NRICH: [Parabolic Patterns](http://nrich.maths.org/773)NRICH: [Tangled Trig Graphs](http://nrich.maths.org/6481)Don Steward: [Graph Transforms](http://donsteward.blogspot.co.uk/search/label/graph%20transforms)**Learning review**KM: [11M5 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M5_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that the graphs of exponential functions y = kx for positive values of k meet or intersect the x-axis.
* Some pupils may think the graph of f(x – 2) is a translation of the graph f(x) by $\left(\begin{matrix}-2\\0\end{matrix}\right)$
* Some pupils may think the graph of f(x) + a is a translation of the graph f(x) by $\left(\begin{matrix}a\\0\end{matrix}\right)$
* Some pupils may think the graph of -f(x) a reflection of the graph f(x) in the y-axis.
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| *Analysing statistics* | *5 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and know their appropriate use
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Construct and interpret histograms
* Analyse distributions of data sets
* Solve problems involving histograms

**Bring on the Maths: GCSE Higher Data**Representing Data: #7Interpreting and Discussing: #9 | * Understand the definition of a histogram
* Construct histograms for grouped data with unequal class intervals
* Use a histogram to find missing values in a frequency table
* Use a partially completed histogram and frequency table to complete both
* Solve problems involving histograms
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the meaning of continuous data
* Understand and use grouped frequency tables
* Interpret histograms for grouped data with equal class intervals
 | Continuous data, Grouped dataTable, Frequency tableFrequencyFrequency densityHistogramScale, GraphAxis, axes**Notation**Correct use of inequality symbols when labeling groups in a frequency table | The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is ‘a diagram made of rectangles whose areas are proportional to the frequency of the group’. If the class widths are equal, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches** |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince Kenny how to construct a histogram with unequal intervals
* What’s the same and what is different: histogram, bar chart?
* Always/Sometimes/Never: The value of the frequency density is less than 1
* Kenny thinks that histogram is just a ‘fancy’ name for a bar chart. Do you agree with Kenny? Explain your answer.
 | KM: What the heck is a histogram?KM: [Stick on the Maths HD3: Working with grouped data](http://www.kangaroomaths.com/free_resources/teaching/sotm/level7/7hd3_ewb.doc)AQA Maths: [Collecting and representing data](http://allaboutmaths.aqa.org.uk/1086)**Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that histogram is a ‘posh term’ for a bar chart
* Some pupils may calculate the frequency density incorrectly such as dividing the bar width by the frequency.
* Some pupils may label the bar of a histogram rather than the boundaries of the bars
* Some pupils may leave gaps between the bars in a histogram
* Some pupils may misuse the inequality symbols when working with a grouped frequency table
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| *Algebraic proficiency: visualising II* | *3 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts)
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Manipulate quadratic functions
* Solve problems involving graphs of quadratic functions
* Explore rates of change

**Bring on the Maths: GCSE Higher Algebra**Solving Quadratic Equations: #2, #12 | * Apply the concept of average rate of change in numerical, algebraic and graphical contexts
* Apply the concept of instantaneous rate of change in numerical, algebraic and graphical contexts
* Solve practical problems involving rates of change
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Complete the square for a given quadratic expression
* Know the meaning of roots, intercepts and turning points
* Identify and interpret roots, intercepts, turning points of quadratic functions graphically
* Interpret the gradient at a point on a curve as the instantaneous rate of change
* Know the effects of transforming the graph y = f(x): f(x) + a and f(x + a)
 | FunctionComplete the squareDeduceRootTurning point, minimum, maximumRate of changeChordTangentAverage rate of changeInstantaneous rate of change**Notation**The form (*x* + *p*)2 – *q* usually implies that completing the square is required | The use of dynamic geometry is essential for this unit to help pupils make connections between the resulting algebraic expression from completing the square and the graph of the quadratic function.This unit provides a good opportunity to reinforce the teaching of transformations of graphs y = f(x): f(x) + a and f(x + a) as explored in the ‘Stage 11 Visualising I’ unit.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils experience dynamic graphing software; e.g. Autograph, throughout this unit* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince Kenny that the co-ordinates of the turning point of y = (x – a)2 – b are (a , -b)
* Always/Sometimes/Never: The y value of the co-ordinates of a turning point of a quadratic function is negative
* Jenny says *‘if you can’t factorise a quadratic then you can’t find the roots of the quadratic algebraically’*. Do you agree with Jenny? Explain your answer.
 | AQA Maths: [Further equations and graphs](http://allaboutmaths.aqa.org.uk/index.php?CurrMenu=1008)AQA Maths: [Sketching Graphs](http://allaboutmaths.aqa.org.uk/1176)AQA Maths: [Gradients and rate of change](http://allaboutmaths.aqa.org.uk/index.php?CurrMenu=1036)Resourceaholic: [Quadratics](http://www.resourceaholic.com/2015/04/tricks-and-tips-3-quadratics.html)Resourceaholic: [Tangents and Areas](http://www.resourceaholic.com/2015/08/graphs.html)**Learning review**KM: [11M4 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/11M4_BAM.pdf)GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may think that the y value of the co-ordinates of a turning point of a quadratic function is always negative.
* Some pupils may think the co-ordinates of the turning point of y = (x – a)2 – b are (-a , -b)
* Some pupils may think the roots of the quadratic y = (x + a)(x + b) are a and b
* Some students may think that (*x* + *p*)2 – *q* implies that *p* must be negative
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| *Mathematical movement II* | *4 lessons* |
| **Key concepts (GCSE subject content statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * use vectors to construct geometric arguments and proofs
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Use vectors to create geometric arguments and proofs

**Bring on the Maths: GCSE Higher Shape**Organising Space: #7 | * Understand how to create and present a proof involving vectors
* Make deductions about situations involving vectors that are multiples of other vectors
* Make deductions about situations involving vectors expressed using ratios
* Make deductions about situations involving vectors and parallel lines
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of a vector
* Use diagrammatic representation of vectors
* Know and use different notations for vectors
* Add and subtract vectors
* Multiply a vector by a scalar
 | VectorScalarConstantMagnitudeCollinear**Notation*****a*** *or* ***a*** (print) and *a* (written) notation for vectors$\vec{AB}$ notation for vectorsColumn vector notation $\left(\begin{matrix}p\\q\end{matrix}\right)$, *p* = movement right and *q* = movement up | In Stage 10, pupils explored how addition, subtraction and multiplication is applied with vectors. This unit involves the use of vectors in geometric arguments and proofs.Vector is a latin word for ‘carrier, transporter’ derived from veho (‘I carry, I transport, I bear’). Vectors have magnitude and direction.Scalar is from the latin ‘scala’ meaning ‘a flight of steps, stairs, staircase’.Scalars have magnitude but no direction.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils either use underline notation, such as a,**or* $\vec{AB}$ *notation when writing vectors* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that ***a*** + ***b*** = ***b*** + ***a***
* Always/Sometimes/Never: $\vec{AB}$ = ***a*** – ***b*** ?
 | AQA Maths: [Bridging the Gap Pocket 9: Vectors](http://allaboutmaths.aqa.org.uk/attachments/5627.pdf)AQA Maths: [Vectors](http://allaboutmaths.aqa.org.uk/index.php?CurrMenu=1074)**Learning review**GLOWMaths/JustMaths: [Sample Questions Higher Tiers](http://justmaths.co.uk/2015/12/21/9-1-exam-questions-by-topic-higher-tier/) | * Some pupils may not appreciate that if a vector is a multiple of another vector, then the two vectors are parallel
* Some pupils may try to write column vectors as fractions, i.e. $\left(\frac{1}{2}\right)$ instead of $\left(\begin{matrix}1\\2\end{matrix}\right)$
* If $\vec{OA}$ = ***a*** and $\vec{OB}$ = ***b***, some pupils may calculate the vector $\vec{AB}$as ***a*** – ***b***
 |