**Primary Mathematics Scheme of Work: Stage 4**

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| *Unit* | *Lessons* | *Key ‘Build a Mathematician’ (BAM) Indicators* | *Essential knowledge* |
| [Numbers and the number system](#NNS) | 8 | * [Round any number to the nearest 10, 100, 1000 and round a number with one decimal place to the nearest whole number](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M1_BAM.pdf)
* [Count backwards through zero](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M2_BAM.pdf)
* [Use columnar addition and subtraction with numbers up to four digits](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M3_BAM.pdf)
* [Multiply two- and three-digit numbers by a one-digit number](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M4_BAM.pdf)
* [Use known and derived facts to multiply and divide mentally](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M5_BAM.pdf)
* [Write any number of tenths or hundredths as a decimal](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M6_BAM.pdf)
* [Find families of common equivalent fractions](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M7_BAM.pdf)
* [Add and subtract fractions with the same denominator](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M8_BAM.pdf)
* [Find areas of rectilinear shapes by counting squares](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M9_BAM.pdf)
* [Use a line of symmetry to complete a symmetric shape or pattern](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M10_BAM.pdf)
* [Identify lines of symmetry in 2D shapes](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M11_BAM.pdf)
* [Use coordinates in the first quadrant](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M12_BAM.pdf)
* [Interpret and construct bar charts and time graphs](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M13_BAM.pdf)
 | * Know the place value headings of ones, tens, hundreds and thousands
* Know the Roman numerals I, V, X, L, C
* [Know multiplication facts up to 12 × 12](http://kangaroomaths.com/free_resources/display/chinese_tables.docx)
* Know division facts related to tables up to 12 × 12
* Know decimals equivalents of 1/2, 1/4, 3/4
* Know adjacent time facts involving years, months, weeks, days, hours, minutes and seconds
* Know 12- and 24-hour clock conversions
* [Know the names and connected properties of triangles](http://kangaroomaths.com/free_resources/display/triangles.pdf) and [quadrilaterals](http://kangaroomaths.com/free_resources/display/quadrilaterals.pdf)
* Know the definitions of acute and obtuse angles
* Know that area is measured in squares
* Know that perimeter is a measure of length
 |
| [Counting and comparing](#CC) | 12 |
| [Investigating properties of shapes](#IPS) | 12 |
| [Calculating: addition and subtraction](#CAS) | 12 |
| [Calculating: multiplication and division](#CMD) | 16 |
| [Exploring time and money](#ETM) | 8 |
| [Exploring fractions, decimals and percentages](#EFDP) | 12 |
| [Measuring space](#MS) | 8 |
| [Investigating angles](#IA) | 8 |
| [Calculating fractions, decimals and percentages](#CFDP) | 8 |
| [Calculating space](#CS) | 8 |
| [Checking, approximating and estimating](#CAE) | 4 |
| [Mathematical movement](#MM) | 8 |
| [Presentation of data](#PD) | 8 |
| Preventing the gap / Going deeper | 8 |  |  |
| Total: | 128 | [Stage 4 BAM Progress Tracker Sheet](http://kangaroomaths.com/free_resources/planning/stage4_tracker.pdf) |  |

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| **Maths Calendar** | *Based on 4 maths lessons per week, with at least 35 'quality teaching' weeks per year*  |
| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 |
| [Numbers & the number system](#NNS) | [Counting and comparing](#CC) | [Investigating properties of shapes](#IPS) | [Calculating: addition and subtraction](#CAS) | [Calc: multiplication and division](#CMD) |
|  | [4M2 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M2_BAM.pdf) | [4M10 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M10_BAM.pdf), [4M11 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M11_BAM.pdf) | [4M3 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M2_BAM.pdf) |  |
| Week 14 | Week 15 | Week 16 | Week 17 | Week 18 | Week 19 | Week 20 | Week 21 | Week 22 | Week 23 | Week 24 | Week 25 | Week 26 |
| [Calc: multiplication and division](#CMD) | Assess / enrich | [Exploring time and money](#ETM) | [Exploring fractions, decimals and percentages](#EFDP) | [Measuring space](#MS) | [Investigating angles](#IA) |
| [4M4 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M4_BAM.pdf), [4M5 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M5_BAM.pdf) |  |  | [4M6 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M6_BAM.pdf) |  |  |
| Week 27 | Week 28 | Week 29 | Week 30 | Week 31 | Week 32 | Week 33 | Week 34 | Week 35 | Week 36 | Week 37 | Week 38 | Week 39 |
| Assess / enrich | [Calculating FDP](#CFDP) | [Calculating space](#CS) | [Checking etc.](#CAE) | [Mathematical movement](#MM) | [Presentation of data](#PD) | Assess / enrich | Preventing the gap / Going deeper |
|  | [4M7 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M7_BAM.pdf), [4M8 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M8_BAM.pdf) | [4M9 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M9_BAM.pdf) | [4M1 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M1_BAM.pdf) | [4M12 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M12) | [4M13 BAM](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M6_BAM.pdf) |  |  |

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| *Numbers and the number system* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
* read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value
* identify, represent and estimate numbers using different representations
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Work with numbers less than 10 000
* Understand and use Roman numerals
* Explore the history of our number system
* Explore ways of representing numbers
* Develop skills of estimation

**Bring on the Maths+: Lower Key Stage 2**Number and Place Value: Place Value II, Roman numerals I, Roman numerals II | * Order numbers up to and including those with four digits
* Write numbers up to and including those with four digits
* Read numbers up to and including those with four digits
* Read Roman numerals up to C
* Understand the difference between the Roman numeral system and the decimal number system
* Interpret numbers up to 10 000 on a number line
* Represent numbers up to 10 000 using a number line
* Use and interpret scales representing measurements with numbers up to 10 000
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand place value in numbers up to three digits
* Know the Roman numerals I, V and X
* Read Roman numerals up to XII
* Use zero as a place holder in two- and three-digit numbers
* Use and interpret a number line to represent numbers
 | Place valueDigitThousandsHundredsTensOnesZeroRoman NumeralEstimateNumber lineScale**Notation**See notes about Roman numerals | Zero is neither positive nor negative.Compare and contrast the Roman number system and Base 10 system – logical structure, place value, role of zero, etc.In general it is incorrect to repeat a Roman numeral symbol four times (i.e. XXXX). Also, the subtractive method should only be used (1) if subtracting powers of ten (i.e. I, X or C), and (2) if subtracting from the next two higher symbols (for example, I can be subtracted from V or X, but not L, C, D or M). Therefore 49 cannot be written as XXXXIX, or as IL, and must be written as XLIX. See NCETM: [Roman numerals](https://www.ncetm.org.uk/resources/11689) NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom displays a number line up to 10 000**Every classroom has a place value chart on the wall* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a four-digit number with a tens unit of ‘6’. And another. And another …
* Kenny writes the number 99 in Roman numerals as IC. Jenny thinks is should be LXXXXVIIII. Do you agree with Kenny or Jenny?
* Benny writes the number two thousand and thirty six as ‘20036’. Do you agree with Benny?
* What’s the same and what’s different: Roman numerals and Base 10 (Denary) number system?

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | NRICH: [Nice or Nasty Games](http://nrich.maths.org/6605)NCETM: [Roman Numerals](https://www.ncetm.org.uk/resources/42476): Activity J**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that zero is positive as it does not have a negative symbol
* Some pupils may use ‘base 10 thinking’ to write Roman numerals, for example 49 as ‘IL’ as it is ‘one less than fifty’
* Some pupils may write four digits numbers literally, for example, four thousand and twenty six as ‘400026’
* Some pupils may ignore place value and simply write the digits mentioned in a number, for example, four thousand and twenty six as ‘426’
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| *Counting and comparing* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * order and compare numbers beyond 1000
* count in multiples of 6, 7, 9, 25 and 1000
* count backwards through zero to include negative numbers
* compare numbers with the same number of decimal places up to two decimal places
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Work with numbers less than 10 000
* Explore ways of counting
* Investigate numbers less than zero
* Compare and order decimals
* Solve problems involving measurement

**Bring on the Maths+: Lower Key Stage 2**Number and Place Value: Counting III | * Order numbers up to 10 000
* Compare numbers up to 10 000
* Count in multiples of 6
* Count in multiples of 7
* Count in multiples of 9
* Count in multiples of 25
* Count in multiples of 1000
* Understand the concept of a negative number
* Count backwards through zero in whole number step
* Compare and order numbers with one decimal place
* Compare numbers with two decimal places
* Order numbers with two decimal places
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand place value in numbers with up to four digits
* Understand place value to tenths
* Use <, > and = symbols
* Count in steps of 4, 8, 50 and 100
 | Place valueDigitMultipleMoreLessZeroPositiveNegative(One, Two) Decimal PlaceNumber line | Zero is neither positive nor negative.It is expected that all pupils should count from 0 in multiples of 6, 7, 9, 25 and 1000, but they should also be given the opportunity to start with any given number.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers use the language ‘negative number’, and not ‘minus number’, to avoid future confusion with calculations**Every classroom has a* [*negative number washing line*](http://kangaroomaths.com/free_resources/display/number_line.docx) *and a place value chart on the wall**Every classroom displays a number line up to 10 000* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * What is the same and what is different: 2*345, 4325, 3425, 4352* ?
* Convince me that 4060 > 4059
* Lenny is counting backwards through zero: 3, 2, 1, 0, -9, -8, -7, …Do you agree with Lenny?
* Jenny thinks that 0.36 > 0.4. Do you agree with Jenny?

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | NRICH: [The Thousands Game](http://nrich.maths.org/2646)NRICH: [Clapping Times](http://nrich.maths.org/5482)NRICH: [Music to My Ears](http://nrich.maths.org/5483/index)NRICH: [Sea Level](http://nrich.maths.org/5929)NCETM: [Counting from a Random Number](https://www.ncetm.org.uk/resources/42476)NCETM: [Rounding and Comparing Decimals](https://www.ncetm.org.uk/resources/42648): Activity G**Learning review**KM: [4M2 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M2_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think the negative number line is:

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 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10* Some pupils think that 1.32 > 1.4 because it has more decimal places
* Some pupils may think that multiples stop at the ‘end of the times tables’, e.g. 84 is the highest multiple of 7.
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| *Investigating properties of shapes* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * identify lines of symmetry in 2-D shapes presented in different orientations
* complete a simple symmetric figure with respect to a specific line of symmetry
* compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore symmetry
* Explore symmetrical patterns
* Investigate 2D shapes

**Bring on the Maths+: Lower Key Stage 2**Properties of shapes: Using properties of shapes, Using a line of symmetry, Identifying symmetry | * Identify and describe an equilateral triangle
* Identify and describe an isosceles triangle
* Identify and describe a scalene triangle
* Identify and describe a parallelogram
* Identify and describe a rhombus
* Identify and describe a trapezium
* Identify and describe a kite
* Classify 2D shapes
* Identify lines of symmetry of a 2D shape
* Identify a line of symmetry of a pattern and for a diagram of a reflection
* Use a line of symmetry to produce a symmetrical pattern
* Use a line of symmetry to complete a symmetrical shape
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Reflect a shape in a vertical line of symmetry
* Use a ruler to construct a straight line joining two points
* Know the names of special quadrilaterals
 | SymmetryLine of symmetry, Mirror lineReflect, ReflectionCongruentPerpendicular, ParallelVertex (Vertices)Side, EdgeQuadrilateralSquare, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, RhombusTriangleScalene, Right-angled, Isosceles, EquilateralPolygon, Hexagon, Pentagon, Octagon, DecagonCircle**Notation**Dash notation to represent equal lengths in shapes and geometric diagramsRight angle notation to indicate perpendicular lines | Pupils should be able to identify a mirror line at 45°, but not to use one to carry out a reflection.Isosceles means ‘equal legs’Encourage pupils to draw irregular polygons, especially hexagons, pentagons, octagons and decagonsNCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)Useful resources: Sets of 2D and 3D shapes, polydron, mirrors, pegboards, pinboards, rulers, dot isometric paper, 2-D shape dice, string**Common approaches***Every classroom has a set of* [*triangle posters*](http://kangaroomaths.com/free_resources/display/triangles.pdf) *and* [*quadrilateral posters*](http://kangaroomaths.com/free_resources/display/quadrilaterals.pdf) *on the wall**Every classroom displays shapes in different orientations.**Every classroom displays regular and irregular hexagons, pentagons, octagons and decagons* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a shape with one line of symmetry, two lines of symmetry. And Another …
* Always/Sometimes/Never: Triangles have three lines of symmetry
* What is the same and what is different ?
* Convince me that a rectangle does not have four lines of symmetry.
* Always/Sometimes/Never: Hexagons have six lines of symmetry

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Naming shapes](http://kangaroomaths.com/free_resources/teaching/geometry/naming_shapes.docx)NRICH: [Stringy Quads](http://nrich.maths.org/2913)NRICH: [National Flags](http://nrich.maths.org/2913)NRICH: [Triangles All Around](http://nrich.maths.org/2850)NRICH: [Symmetry Challenge](http://nrich.maths.org/1886)**Learning review**KM: [4M10 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M10_BAM.pdf), [4M11 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M11_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think a rectangle has four lines of symmetry
* Some pupil may think that a triangle always has to have a horizontal base:
* Some pupils think that all hexagons, pentagons, octagons and decagons are regular.
* Some pupils think that a rhombus is a square ‘standing’ on one of its edges.
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| *Calculating: addition and subtraction* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * find 1000 more or less than a given number
* add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
* solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop mental methods of addition and subtraction
* Extend written methods of addition and subtraction
* Solve problems involving addition and subtraction

**Bring on the Maths+: Moving on up!**Calculating: v1**Bring on the Maths+: Lower Key Stage 2**Calculating: Addition and subtraction using written methods II | * Find 1000 more than a given number
* Find 1000 less than a given number
* Use columnar addition for numbers with up to four digits with no carrying required
* Use columnar addition for four-digit and two-digit numbers with carrying required
* Use columnar addition for four-digit and three-digit numbers with carrying required
* Use columnar addition for four-digit numbers with carrying required
* Use columnar subtraction for numbers with up to four digits with no exchanging required
* Use columnar subtraction for four-digit and two-digit numbers with exchanging required
* Use columnar subtraction for four-digit and three-digit numbers with exchanging required
* Use columnar subtraction for four-digit and four-digit numbers with exchanging required
* Solve two-step problems involving addition and/or subtraction
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Find 100 more or less than a given number
* Use column addition and subtraction for numbers up to three digits
 | AdditionSubtractionSum, TotalDifference, Minus, LessColumn additionColumn subtractionExchangeOperationEstimate | Ensure that pupils can deal with column subtractions that include a 0 within the first number; e.g. 8027 – 437.Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.KM: [Progression: Addition and Subtraction](http://kangaroomaths.com/free_resources/hod/bouncebuzz_addition_subtraction_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44567), [Subtraction](https://www.ncetm.org.uk/resources/40532)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition. Teachers say ‘complete these calculations’ instead of ‘complete these sums’**All pupils use books / paper with 1cm squares and ensure that each digit is written in one square**When carrying, those numbers being carried are placed beneath the answer line**During column subtraction the language of ‘exchanging’ is used instead of ‘borrowing’**When exchanging, those numbers being altered or moved are written above the calculation*  |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Provide examples of column addition and subtraction with hidden digits. Challenge pupils to find these digits and explain their reasoning.
* Show me an example of a column addition (that includes carrying) with the answer 2106
* Convince me that 6095 – 3622 = 2473

NCETM: [Addition and Subtraction Reasoning](https://www.ncetm.org.uk/public/files/18416326/2_Progression_Map_Addition_and_Subtraction_Reasoning.pdf) | KM: [Interactive target boards](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_Interactive_Target_Boards.xlsm)KM: [Maths to Infinity: Addition and subtraction foundations](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_TheFoundations_AddSub.xlsm)NRICH: [Dicey operations](http://nrich.maths.org/6606): Game 2NCETM: [Interactive Base 10 Blocks](https://www.ncetm.org.uk/resources/42543): Activity ANCETM: [Images of addition and subtraction](https://www.ncetm.org.uk/resources/42543): Activity C**Learning review**KM: [4M3 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M3_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils incorrectly assume and use commutativity within column subtraction; for example:

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| --- | --- | --- | --- | --- |
|  | 4 | 1 | 2 | 6 |
| – | 3 | 7 | 3 | 4 |
|  | 1 | 6 | 1 | 2 |

* Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)
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| *Calculating: multiplication and division* | *16 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * recall multiplication and division facts for multiplication tables up to 12 × 12
* recognise and use factor pairs and commutativity in mental calculations
* use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
* multiply two-digit and three-digit numbers by a one-digit number using formal written layout
* solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop mental arithmetic skills
* Develop knowledge of multiplication tables
* Explore place value
* Develop written methods of multiplication
* Solve problems involving multiplication and division

**Bring on the Maths+: Lower Key Stage 2**Number and Place Value: Place value and calculations, Number patterns: divisibility testingCalculating: Deriving facts from known facts, Multiplying and dividing, Using tablesTimes tables: The six times table, The seven times table, The nine times table, the eleven times table, The twelve times table, All mixed up | * Recall and use multiplication facts for the 6 times table
* Recall and use multiplication facts for the 7 times table
* Recall and use multiplication facts for the 9 times table
* Recall and use multiplication facts for the 11 times table
* Recall and use multiplication facts for the 12 times table
* Recall and use division facts for the 6 times table
* Recall and use division facts for the 7 times table
* Recall and use division facts for the 9 times table
* Recall and use division facts for the 11 times table
* Recall and use division facts for the 12 times table
 | * Use knowledge of factor pairs (commutativity) when multiplying and dividing mentally including multiplying three numbers together
* Know the effect of multiplying by 0 and 1 and dividing by 1
* Use the distributive law to multiply a two-digit number by a one-digit number
* Use short multiplication to multiply a two-digit number by a one-digit number
* Use short multiplication to multiply a three-digit number by a one-digit number
* Identify when a scaling or correspondence problem can be solved using multiplication or division
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recall multiplication and division facts for 2, 3, 4, 5, 8 and 10 multiplication tables
* Understand that multiplication and division are inverse operations
 | Mental arithmeticPlace valueMultiply, Multiplication, Times, ProductCommutativeDivide, DivisionTenth, HundredthFactor, Factor pairsShort multiplicationOperationEstimate | Pupils make the connection between arrays and the compact grid method. This is promoted as a method that aids conceptual understanding of short and long multiplication and progress to multiplying algebraic expressions.Later in this stage there is further opportunity to develop calculation skills with an emphasis on checking, approximating or estimating.KM: [Progression: Multiplication and Division](http://kangaroomaths.com/free_resources/hod/bouncebuzz_multiplication_division_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44568), [Multiplication](https://www.ncetm.org.uk/resources/40530) , [Division](https://www.ncetm.org.uk/resources/43589), [Multiplicative reasoning](https://www.ncetm.org.uk/resources/43669), [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All classrooms display a* [*times table poster with a twist*](http://kangaroomaths.com/free_resources/display/chinese_tables.docx)*Teachers say ‘complete these calculations’ instead of ‘complete these sums’. Short division is promoted as the ‘most efficient method’.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Provide examples of multiplication of two-digit and three-digit numbers by a one-digit number using formal written layout with missing digits. Challenge pupils to find these digits and explain their reasoning.
* A × B × C = 120. Show me possible values for A, B and C. And another triple. And another triple.
* Always/Sometimes/Never: *‘When you multiply two number together, the answer is greater than both of the two numbers’.*

NCETM: [Multiplication and Division Reasoning](https://www.ncetm.org.uk/public/files/18438909/3_Progression_Map_Multiplication_and_Division_Reasoningv2.pdf) | KM: [Interactive target boards](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_Interactive_Target_Boards.xlsm)KM: [Maths to Infinity: Multiplication and division foundations](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_The%20Foundations_MultDiv.xlsm)NRICH: [Multiplication Square Jigsaw](http://nrich.maths.org/5573) ,[Mystery matrix](http://nrich.maths.org/public/viewer.php?obj_id=1070), [Abundant Numbers](http://nrich.maths.org/1011)NCETM: [Models and Images (Number Trios)](https://www.ncetm.org.uk/resources/42543): Activity DNCETM: [12 x 12 Activities](https://www.ncetm.org.uk/resources/42598): Activity ANCETM: [Multiplying and Dividing with Straws](https://www.ncetm.org.uk/resources/42598): Activity BNCETM: [sdMultilink Multiplication](https://www.ncetm.org.uk/resources/42598): Activity D**Learning review:**KM: [4M4 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M4_BAM.pdf), [4M5 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M5_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write statements such as 2 ÷ 8 = 4
* Some pupils may carry the wrong digit when using short multiplication; for example:

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| *Exploring time and money* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * read, write and convert time between analogue and digital 12- and 24-hour clocks
* solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days
* estimate, compare and calculate different measures, including money in pounds and pence
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve problems involving time
* Solve problems involving money
* Estimate measures

**Bring on the Maths+: Lower Key Stage 2**Measures: Time facts, The 24 hour clock | * Read digital 24-hour clocks
* Write times using digital 24-hour clock
* Write times using analogue 12-hour clock
* Convert between 12-hour time and 24-hour notation
* Solve problems involving converting from hours to minutes and minutes to seconds;
* Solve problems involving converting from weeks to days
* Solve problems involving converting from years to months
* Solve problems involving decimal notation to record money
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use analogue and digital 12-hour clocks
* Know the number of seconds in a minute, minutes in an hour, hours in a day, and the number of days in each week, month, year and leap year
* Know the value of all British coins and notes
* Know the number of pence in a pound
* Calculate the duration of time for a given event or task
 | AnalogueDigital12-hour24-hourSecond, Minute, HourDay, Week, Month, YearPound (£)Pence (p)LengthMassVolume**Notation**£ and p12-hour and24-hour notation use a ‘:’, for example 18:40 and 9:30 a.m. | Some Jewish clocks rotate anti-clockwise. There is a 24-hour analogue clock at Greenwich observatory.24 clock notation using four digits. Any time before 10:00 a.m. uses a zero as the second hour digit, for example 9:15 a.m. is written as ‘09:15’.Noon is 12:00 and midnight is 00:00NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Use of a colon to write 12- and 24- hour times**Noon is treated as 12:00 and midnight as 00:00**Use either the ‘£’ or ‘p’ symbol but not both symbols at the same time, for example, £4.56p* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Compare quantities by estimation
* Ask students to tell the time of using images of the Greenwich Observatory Clock or 24 Hour watches. How does a 24-hour watch need to operate differently to a 12-hour watch?
* Convince me that 18:40 is the same as 6:40 pm.
* Convince me that ‘two pounds and five pence’ is not written as ‘£2.5p’

NCETM: [Measurement Reasoning](https://www.ncetm.org.uk/public/files/18436766/7_Progression_Map_Measurement_Reasoning.pdf) | NRICH: [Wonky Watches](http://nrich.maths.org/public/viewer.php?obj_id=1002)NRICH: [Five Coins](http://nrich.maths.org/142)NRICH: [Money Measure](http://nrich.maths.org/2417)NRICH: [Pouring the Punch Drink](http://nrich.maths.org/965)NCETM: [Worms](https://www.ncetm.org.uk/resources/42725): Activity A**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils do not write 24-hour times as four digits, for example 1:50 rather than 01:50 for 1.50 a.m.
* Some pupils may write ‘one pound and eight pence’ as ‘£1.8’
* Some pupils may use both ‘£’ and ‘p’ symbols, for example ‘£4.56p’
* Some pupils may write ‘four pounds and fifty six pence’ as ‘4.56p’ or ‘£456’
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| *Exploring fractions, decimals and percentages* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
* find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
* recognise and write decimal equivalents of any number of tenths or hundredths
* recognise and write decimal equivalents to 1/4, 1/2, 3/4
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop knowledge of place value
* Explore decimals
* Investigate fractions and decimals

**Bring on the Maths+: Lower Key Stage 2**Fractions & Decimals: Simple fractions, Decimal notation, Equivalent fractions, Fractions and decimals I, Fractions and decimals II | * Recognise that hundredths arise from dividing a number or object into one hundred equal parts
* Write hundredths as a fraction and as a decimal
* Write decimal equivalents of any number of tenths and hundredths
* Count up in hundredths
* Count down in hundredths
* Divide a one-digit number by 10
* Divide a one-digit number by 100
* Divide a two-digit number by 10
* Divide a two-digit number by 100
* Know and use the decimal equivalents to 1/4, 1/2, 3/4
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recognise and use tenths
* Divide one digit numbers by 10
 | Place valueTenth, hundredthDecimalDivideFractionNumeratorDenominatorTenthHundredthDecimal**Notation**Decimal pointt, h notation for tenths, hundredths | Pupils need to know the place value headings of tenths and hundredths and recognise that hundredths arise from dividing tenths by tenSome countries use a comma for a decimal point.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils are expected to use horizontal bar notation for fractions**Every classroom has a fraction / decimal number line on display**Every classroom has a place value chart on display* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Jenny is counting in hundredths ‘…. 2.07, 2.08, 2.09, 2.010, 2.011 …’. Do you agree with Jenny? Explain your answer.
* Convince me 0.17 = $\frac{17}{100}$
* Kenny thinks the decimal 0.8 can be written as $\frac{80}{100}$. Lenny thinks the decimal 0.8 can be written as $\frac{8}{10}$. Who do you agree with? Explain your thinking.
* Always/Sometimes/Never: To divide a number by 10, remove the zero.
* Convince me $\frac{1}{4}$ = 0.25

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf) | NRICH: [Fair Feast](http://nrich.maths.org/public/viewer.php?obj_id=2361)NCETM: [Activity B – Tenths and Hundredths](https://www.ncetm.org.uk/resources/42648)NCETM: [Activity A – interactive programmes](https://www.ncetm.org.uk/resources/42648)NCETM: [Activity E – decimal equivalents](https://www.ncetm.org.uk/resources/42648)**Learning review**KM: [4M6 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M6_BAM.pdf),NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that the first place value heading after the decimal point is ‘one-ths’ /‘unit-ths’ and the second place value heading is ‘tenths’.
* Some pupils may read 0.25 as ‘nought point twenty five’
* Some pupils may think that 1/4 = 1.4
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| *Measuring space* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * convert between different units of measure [for example, kilometre to metre; hour to minute]
* solve simple measure and money problems involving fractions and decimals to two decimal places
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Convert between measures
* Solve problems involving measurement
* Solve problems involving money

**Bring on the Maths+: Lower Key Stage 2**Measures: Measures | * Convert between kilometres and metres
* Convert between centimetres and millimetres
* Convert between kilograms and grams
* Convert between litres and millilitres
* Solve measurement problems involving fractions
* Solve money problems involving fractions
* Solve measurement problems involving decimals to two decimal places
* Solve money problems involving decimals to two decimal places
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use a ruler to measure lengths to the nearest millimetre
* Use digital and mechanical scales to measure mass
* Use measuring vessels to measure a volume of liquid
* Choose appropriate units to state the result of a measurement
* Compare the length (mass, volume, capacity) of two or more objects
* Solve measurement problems involving addition or subtraction
 | Length, distanceMassVolumeCapacityMetre, centimetre, millimetreKilogram, gramLitre, millilitreHour, minute, secondDecimal**Notation**Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml | Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.The prefix ‘centi-‘ means one hundredth, and the prefix ‘milli-‘ means one thousandth. These words are of Latin origin.The prefix ‘kilo-‘ means one thousand. This is Greek in origin.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a length in cm that is between 25mm and 0.5m. And another. And another.
* Which is greater, 4.5 litres or 460 ml? Explain your answer.
* Kenny thinks that 120 minutes is the same as 1 hour and 20 minutes. Do you agree with Kenny? Explain your answer.
* Convince me that 15:00 = 3 p.m.

NCETM: [Measurement Reasoning](https://www.ncetm.org.uk/public/files/18436766/7_Progression_Map_Measurement_Reasoning.pdf) | NRICH: [Discuss and Choose](http://nrich.maths.org/7449)NCETM: [Activity Set A](https://www.ncetm.org.uk/resources/42733)NCETM: [Activity Set C](https://www.ncetm.org.uk/resources/42733)NCETM: [Activity Set D](https://www.ncetm.org.uk/resources/42733)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
* Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
* Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’ clock
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| *Investigating angles* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * identify acute and obtuse angles and compare and order angles up to two right angles by size
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop knowledge of angles
* Compare angles

**Bring on the Maths+: Lower Key Stage 2**Properties of Shapes: Identifying angles | * Identify acute angles
* Identify obtuse angles
* Identify acute angles in shapes
* Identify obtuse angles in shapes
* Identify right angles in shapes
* Compare angles up to two right angles in size
* Order angles up to two right angles in size
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand angles as a measure of turn
* Recognise angles in shapes
* Identify right angles as a quarter turn
 | TurnAngleRight angleAcute angleObtuse angleGreater than, less than**Notation**Right angle notationArc notation for all other angles | Understanding degrees as a way of measuring angles is not introduced until Stage 5.It is thought that the origin of the name ‘right angle’ is the Latin word for ‘upright’; as in perpendicular to the horizontal base in architectural contexts.Acute is from the Latin word for ‘sharp’, acutusObtuse is from the Latin word for ‘blunt’ or ‘dull’, obtususAngle is from the Latin word for corner, ‘angulusNCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils experience the ‘feel’ of acute and obtuse angles by turning on the spot* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an acute (obtuse) angle. And another. And another.
* (With your arms), show me an angle greater (less) than a quarter turn. And another. And another.
* Kenny thinks that a quarter turn is an acute angle. Jenny thinks that a quarter turn is an obtuse angle. Who is correct? Explain your answer.
* What is the same and what is different:

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | NRICH: [Triangles All Around](http://nrich.maths.org/2850)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils think that a quarter turn is an acute angle; some may think it’s an obtuse angle.
* Some pupils may think that one part of a drawn angle must be horizontal
* Some pupils may think that all turns have to be in a clockwise direction
* Some pupils may think that a drawn angle must use two lines of equal length
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| *Calculating fractions, decimals and percentages* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * add and subtract fractions with the same denominator
* solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
* recognise and show, using diagrams, families of common equivalent fractions
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate with fractions
* Investigate equivalent fractions

**Bring on the Maths+: Lower Key Stage 2**Fractions and decimals: Adding and subtracting fractions | * Add fractions with the same denominator within and beyond one whole
* Subtract fractions with the same denominator within and beyond one whole
* Calculate a unit fraction of an amount when the answer is a whole number
* Calculate a non-unit fraction of an amount when the answer is a whole number
* Identify equivalent fractions from diagrams
* Find families of equivalent fractions
* Create diagrams to show families of equivalent fractions
* Solve problems with increasingly harder fractions to calculate quantities
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Add and subtract fractions with the same denominator within one whole
* Calculate fractions such as 1/2 of 6 = 3
* Understand the concept of equivalent fractions
* Recognise equivalent fractions from diagrams
* Complete diagrams to show equivalent fractions
 | FractionUnit fraction, non-unit fractionImproper fractionTop-heavy fractionNumerator, denominatorAdd, subtractEquivalent (fraction)Family**Notation**Horizontal bar for fractionsDiagonal bar for fractions | Describe 1/3 as ‘there are three equal parts and I take one’, and 3/4 as ‘there are four equal parts and I take three’.Be alert to pupils reinforcing misconceptions through language such as ‘the bigger half’.To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded.When adding and subtracting fractions beyond one whole pupils are only expected to work with improper fractions (and not mixed numbers)NCETM: [Teaching fractions](https://www.ncetm.org.uk/resources/44490)NCETM: [Fractions videos](https://www.ncetm.org.uk/resources/43609)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44565)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers use the language ‘improper fraction’ and not ‘top-heavy fraction’**When finding fractions of an amount pupils work denominators of at least 2 to 10* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a fraction equivalent to $\frac{3}{4}$ that no one else will think of. And another. And another
* Show me two fractions that add together to make a whole. And another pair. And another pair.
* Show me one improper and one proper fraction such that the difference is less than one whole. And another pair. And another pair.
* Kenny thinks that ¾ of 24 is 2 because 24 ÷ 4 = 6 and 6 ÷ 3 = 2. Do you agree with Kenny? Explain your answer.
* Jenny think that $\frac{16}{64}$ = $\frac{1}{4}$ because your can simply cancel the ‘6’ in the numerator and denominator. Do you agree with Jenny’s method for finding equivalent fractions? Explain your answer

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf) | NRICH: [Fair Feast](http://nrich.maths.org/2361)NRICH: [Fractional Triangles](http://nrich.maths.org/2124)NCETM: [Activity D – adding and subtracting fractions](https://www.ncetm.org.uk/resources/42648)NCETM: [Activity C – fractions to calculate quantities](https://www.ncetm.org.uk/resources/42648)**Learning review**KM: [4M7 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M7_BAM.pdf)KM: [4M8 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M8_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that you simply add/subtract the numerators and add/subtract the denominators when adding fractions.
* Some pupils may think that you find the non-unit fraction of an amount by dividing by the denominator (as with unit fractions) and then dividing by the numerator. They do not make the connection that ¾ = 3 x ¼.
* Some pupils may think that diagrams to show fractions must always be circular.
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| *Calculating space* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
* find the area of rectilinear shapes by counting squares
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate perimeter
* Investigate area

**Bring on the Maths+: Lower Key Stage 2**Measures: Perimeter, Area and perimeter | * Measure and calculate the perimeter of 2D shapes when dimensions are unknown
* Calculate the perimeter of rectangles (including squares) when dimensions are known
* Calculate the perimeter of other rectilinear shapes when dimensions are known
* Find the area of rectangles (including squares) by counting squares
* Find the area of other rectilinear shapes by counting squares
* Solve problems involving perimeter
* Solve problems involving area
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Find the perimeter of a simple 2D shape by measuring
 | PerimeterAreaDimensionsSquareRectangleRectilinearPolygonMillimetre, Centimetre, Metre, Kilometre**Notation**Abbreviations of units in the metric system: km, m, cm, mm | For the purposes of this unit, rectilinear shapes are those that are bounded by straight lines. This includes rectangles and shapes made from rectangles and triangles, see NCETM: [Y4 Measurement exemplification](https://www.ncetm.org.uk/resources/42734)This unit covers two concepts that pupils often confuse. It would be unwise to have a single objective for a lesson (or lessons) covering both area and perimeter.Note that at this stage area is measured in squares. Using cm2 and m2 is developed in Stage 5.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***When calculating perimeter all pupils experience practical approaches outside the classroom such as walking around the football pitch.*  |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always / Sometimes / Never: perimeter is given by multiplying the shortest side by 2, the longest side by 2, and then adding the results
* Jenny thinks that the perimeter of this rectangle is 9cm. Do you agree with Jenny?

 3cm 6cm* Create a shape with a perimeter greater then 30 m

NCETM: [Geometry -Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Stick on the Maths SSM6: Area and perimeter](http://www.kangaroomaths.com/free_resources/teaching/sotm/level4/4ssm6_ewb.doc)NRICH: [Torn Shapes](http://nrich.maths.org/public/viewer.php?obj_id=4963)NCETM: [Activity B](https://www.ncetm.org.uk/resources/42733)**Learning review**KM: [4M9 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M9_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may find perimeters by counting the squares that are around the outside of the shape, rather than the associated lengths. This will result in an answer four less than the correct answer if working with rectangles.
* Some pupils may confuse the concepts of area and perimeter
* Some pupils may think that perimeter is always given by 2(a + b) or 2a + 2b, but this is only the case for certain 2D shapes
* Some pupils may think that you multiply the numbers to find the perimeter of a shape.
* Some pupils may think that you cannot find the perimeter of a shape unless all the dimensions are given.
* Some pupils may just add the given dimensions, rather than consider any unlabelled dimensions
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| *Checking, approximating and estimating* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * round any number to the nearest 10, 100 or 1000
* round decimals with one decimal place to the nearest whole number
* estimate and use inverse operations to check answers to a calculation
* solve number and practical problems that involve all of the above and with increasingly large positive numbers
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore ways of approximating numbers
* Explore ways of checking answers

**Bring on the Maths+: Moving on up!**Number and Place Value: v2**Bring on the Maths+: Lower Key Stage 2**Number and Place Value: Place value and approximations | * Approximate any number by rounding to the nearest 10, 100 or 1000
* Approximate any number with one decimal place by rounding to the nearest whole number
* Understand checking as the process of working backwards from the answer to ensure that it makes sense
* Understand estimating as the process of finding a rough value of an answer or calculation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Order and compare numbers up to 10 000
* Order and compare numbers with one decimal place
* Know that addition and subtraction are inverses of each other
* Know that multiplication and division are inverses of each other
 | Approximate (noun and verb)RoundDecimal placeCheckSolutionAnswerEstimate (noun and verb)**Notation** The approximately equal symbol (≈) | This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.Also see big pictures: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) and [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to visualise rounding through the use a number line* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that 15 rounds to 20 to the nearest 10
* What is the same and what is different: 595, 649, 534 and 634
* Kenny thinks that 4.6 rounds to 4 to the nearest whole number. Do you agree? Explain your answer.

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | KM: [Up or down?](http://kangaroomaths.com/free_resources/teaching/number/up_or_down.docx)KM: [Maths to Infinity Rounding](http://www.kangaroomaths.com/free_resources/infinity/rounding.xlsm)NCETM: [Activity I – Rounding Quiz](https://www.ncetm.org.uk/resources/42476)**Learning review**KM: [4M1 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M1_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may truncate instead of round
* Some pupils may misunderstand the rounding process as one that works from the end of the number; for example 347 to the nearest 100 is worked out as 347 🡪 350 🡪 400.
* Some pupils may round down at the half way point, rather than round up.
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| *Mathematical movement* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * describe positions on a 2-D grid as coordinates in the first quadrant
* plot specified points and draw sides to complete a given polygon
* describe movements between positions as translations of a given unit to the left/right and up/down
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use Cartesian coordinates
* Use transformations to move shapes

**Bring on the Maths+: Moving on up!**Position and direction: v2**Bring on the Maths+: Lower Key Stage 2**Position and direction: Coordinates in the first quadrant | * Use coordinates to describe the position of a point in the first quadrant
* Plot points in the first quadrant using co-ordinates
* Use coordinates to plot a set of points to construct a polygon
* Solve problems involving coordinates
* Describe movements between positions as translations of a given unit to the left/right
* Describe movements between positions as translations of a given unit to the up/down
* Describe movements between positions as translations of a given unit to the left/right and up/down
* Solve problems involving translation
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know names and basic properties of polygons
* Know the language of movement; left, right, up and down
 | 2-DGridAxis, axes, x-axis, y-axisOrigin(First) quadrant(Cartesian) coordinatesPointTranslationTransformationLeft, right, up, down**Notation**Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) | The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.Other coordinate systems include grid references, polar coordinates and spherical coordinates.There are many other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is ‘transformations’.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers do not use the phrase ‘along the corridor and up the stairs’ as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is ‘x is a cross, wise up!’* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Given a grid with the point (2, 3) indicated) Benny describes this point as (2, 3). Jenny describes the point as (3, 2). Who do you agree with? Why?
* Convince me that the coordinates of the origin are (0, 0)
* Always / Sometimes / Never: A translation moves a shape further away from the origin.

NCETM: [Geometry: Position Direction and Movement Reasoning](https://www.ncetm.org.uk/public/files/18436990/9_Progression_Map_Geometry_position_direction_and_movement_Reasoning.pdf) | KM: [Stick on the Maths](http://www.kangaroomaths.com/free_resources/teaching/sotm/level4/4alg2_ewb.doc) [ALG2: Coordinates in the first quadrant](http://www.kangaroomaths.com/free_resources/teaching/sotm/level4/4alg2_ewb.doc)NRICH: [Coordinate Challenge](http://nrich.maths.org/5038)NRICH: [A Cartesian Puzzle](http://nrich.maths.org/5038)NCETM: [Activity C: Translation or Destination 1](https://www.ncetm.org.uk/resources/42938) **Learning review**KM: [4M12 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M12_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
* Some pupils may muddle left and right
* Some pupils will confuse the order of x-coordinates and y-coordinates
* When constructing axes, some pupils may not realise the importance of equal divisions on the axes
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| *Presentation of data* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs
* solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore ways of presenting data
* Solve problems involving charts and graphs

**Bring on the Maths+: Moving on up!**Statistics: v1**Bring on the Maths+: Lower Key Stage 2**Statistics: Interpreting information | * Interpret a pictogram where the symbol represents multiple items
* Interpret a bar chart
* Interpret bar charts with different scales on the frequency axis
* Create a bar chart with different scales on the frequency axis
* Interpret a time graph
* Create a time graph
* Solve problems involving the data in charts and graphs
* Solve problems involving the data in tables
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Interpret and construct a pictogram where the symbol represents multiple items
* Interpret and construct a simple bar chart where one centimetre represents 2, 5 or 10 items
* Interpret and construct tables of data
 | DataPictogramSymbolKeyTallyBar chartTime graphScaleAxisGraphFrequency | William Playfair, a Scottish engineer and economist, introduced the time graph in 1786.Pupils are expected to understand and use more complex scales, i.e. not just 2, 5, or 10 units per cm for bar charts and 2, 5, or 10 units per symbol for pictograms.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils experience bar charts with horizontal and vertical bars* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a time graph of your day and tell me a story about it. And another. And another.
* Penny draws a bar chart with horizontal bars. Benny says the bars must be vertical. Who is correct? Explain your answer.
* Always/Sometimes/Never**:** One centimetre on the frequency axis of a bar chart represents one unit.

NCETM: [Statistics Reasoning](https://www.ncetm.org.uk/public/files/18437062/10_Progression_Map_Statistics_Reasoning.pdf) | KM: [Stick on the Maths HD2: Bar charts and pictograms](http://www.kangaroomaths.com/free_resources/teaching/sotm/level3/3hd2_ewb.doc)KM: [Stick on the Maths HD4: Interpreting information](http://www.kangaroomaths.com/free_resources/teaching/sotm/level3/3hd4_ewb.doc)NRICH: [You tell the story](http://nrich.maths.org/4802)NRICH: [You tell the story](http://nrich.maths.org/4802)NCETM: [Activity A](https://www.ncetm.org.uk/resources/42962)NCETM: [Activity B](https://www.ncetm.org.uk/resources/42962)**Learning review**KM: [4M13 BAM Task](http://www.kangaroomaths.com/free_resources/assessment/BAM/4M13_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may interpret bar charts as one unit of frequency for each one square on the paper used.
* Some pupils may not leave gaps between the bars in a bar chart
* Some pupils may think that one centimetre on the frequency axis of a bar chart always represents one unit in a bar chart.
* Some pupils may think that a symbol always represents one unit in a pictogram.
* Some pupils may think that the bars of a bar chart must be vertical.
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