**Primary Mathematics Scheme of Work: Stage 6**

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| *Unit* | *Lessons* | *Key ‘Build a Mathematician’ (BAM) Indicators* | *Essential knowledge* |
| [Numbers and the number system](#NNS) | 12 | * [Multiply and divide numbers with up to three decimal places by 10, 100, and 1000](http://kangaroomaths.com/free_resources/assessment/BAM/6M1_BAM.pdf)
* [Use long division to divide numbers up to four digits by a two-digit number](http://kangaroomaths.com/free_resources/assessment/BAM/6M2_BAM.pdf)
* [Use simple formulae expressed in words](http://kangaroomaths.com/free_resources/assessment/BAM/6M3_BAM.pdf)
* [Generate and describe linear number sequences](http://kangaroomaths.com/free_resources/assessment/BAM/6M4_BAM.pdf)
* [Use simple ratio to compare quantities](http://kangaroomaths.com/free_resources/assessment/BAM/6M5_BAM.pdf)
* [Write a fraction in its lowest terms by cancelling common factors](http://kangaroomaths.com/free_resources/assessment/BAM/6M6_BAM.pdf)
* [Add and subtract fractions and mixed numbers with different denominators](http://kangaroomaths.com/free_resources/assessment/BAM/6M7_BAM.pdf)
* [Multiply pairs of fractions in simple cases](http://kangaroomaths.com/free_resources/assessment/BAM/6M8_BAM.pdf)
* [Find percentages of quantities](http://kangaroomaths.com/free_resources/assessment/BAM/6M9_BAM.pdf)
* [Solve missing angle problems involving triangles, quadrilaterals, angles at a point and angles on a straight line](http://kangaroomaths.com/free_resources/assessment/BAM/6M10_BAM.pdf)
* [Calculate the volume of cubes and cuboids](http://kangaroomaths.com/free_resources/assessment/BAM/6M11_BAM.pdf)
* [Use coordinates in all four quadrants](http://kangaroomaths.com/free_resources/assessment/BAM/6M12_BAM.pdf)
* [Calculate and interpret the mean as an average of a set of discrete data](http://kangaroomaths.com/free_resources/assessment/BAM/6M13_BAM.pdf)
 | * Know percentage and decimal equivalents for fractions with a denominator of 2, 3, 4, 5, 8 and 10
* Know the rough equivalence between miles and kilometres
* Know that vertically opposite angles are equal
* [Know that the area of a triangle = base × height ÷ 2](http://kangaroomaths.com/free_resources/display/areas.pdf)
* [Know that the area of a parallelogram = base × height](http://kangaroomaths.com/free_resources/display/areas.pdf)
* Know that volume is measured in cubes
* Know the names of parts of a circle
* Know that the diameter of a circle is twice the radius
* Know the conventions for a 2D coordinate grid
* [Know that mean = sum of data ÷ number of pieces of data](http://kangaroomaths.com/free_resources/display/statistics.pdf)
 |
| [Checking, approximating and estimating](#CAE) | 4 |
| [Calculating](#CALC) | 8 |
| [Calculating: division](#CALCD) | 8 |
| [Visualising and constructing](#VC) | 8 |
| [Investigating properties of shapes](#IPS) | 8 |
| [Algebraic proficiency: using formulae](#APF) | 4 |
| [Exploring fractions, decimals and percentages](#EFDP) | 8 |
| [Proportional reasoning](#PR) | 8 |
| [Pattern sniffing](#PS) | 4 |
| [Measuring space](#MS) | 4 |
| [Investigating angles](#IA) | 4 |
| [Calculating fractions, decimals and percentages](#CFDP) | 12 |
| [Solving equations and inequalities](#SEI) | 4 |
| [Calculating space](#CS) | 8 |
| [Mathematical movement](#MM) | 4 |
| [Presentation of data](#PD) | 4 |
| [Measuring data](#MD) | 4 |
| Secondary transition | 24 |  |  |
| Total: | 140 | [Stage 6 BAM Progress Tracker Sheet](http://kangaroomaths.com/free_resources/planning/stage6_tracker.pdf) |  |

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| **Maths Calendar** | *Based on 4 maths lessons per week, with at least 35 'quality teaching' weeks per year*  |
| Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 | Week 13 |
| [Numbers and the number system](#NNS) | [Checking etc](#CAE) | [Calculating](#CALC) | [Calculating: division](#CALCD) | [Visualising and constructing](#VC) | [Inv. properties of shapes](#IPS) | [Formulae](#APF) |
| [6M1 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M1_BAM.pdf) |  |  | [6M2 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M2_BAM.pdf) |  |  | [6M3 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M3_BAM.pdf) |
| Week 14 | Week 15 | Week 16 | Week 17 | Week 18 | Week 19 | Week 20 | Week 21 | Week 22 | Week 23 | Week 24 | Week 25 | Week 26 |
| Assess / enrich | [Exploring FDP](#EFDP) | [Proportional reasoning](#PR) | [Patterns](#PS) | [Meas. space](#MS) | [Angles](#IA) | [Calculating fractions, decimals and percentages](#CFDP) | [Equations](#SEI) | [Calc space](#CS) |
|  | [6M6 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M6_BAM.pdf) | [6M5 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M5_BAM.pdf) | [6M4 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M4_BAM.pdf) |  | [6M10 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M10_BAM.pdf) | [6M7 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M7_BAM.pdf), [6M8 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M8_BAM.pdf), [6M9 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M9_BAM.pdf) |  | [6M11 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M11_BAM.pdf) |
| Week 27 | Week 28 | Week 29 | Week 30 | Week 31 | Week 32 | Week 33 | Week 34 | Week 35 | Week 36 | Week 37 | Week 38 | Week 39 |
| [Calc space](#CS) | [Movement](#MM) | [Pres. data](#PD) | [Meas. data](#MD) | NCT week | Enrichment | Secondary transition |
| [6M11 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M11_BAM.pdf) | [6M12 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M12_BAM.pdf) |  | [6M13 BAM](http://kangaroomaths.com/free_resources/assessment/BAM/6M13_BAM.pdf) |  |  |  |

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| *Numbers and the number system* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
* read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
* use negative numbers in context, and calculate intervals across zero
* identify common factors, common multiples and prime numbers
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use decimals with up to three decimal places
* Work with numbers up to ten million
* Explore the use of negative numbers
* Develop understanding of factors and multiples
* Investigate prime numbers

**Bring on the Maths+: Moving on up!**Number and Place Value: #3, #6Calculating: #2 | * Write and read numbers up to and including 10 000 000
* Compare and order numbers up to and including 10 000 000
* Multiply numbers by 10
* Multiply numbers by 100
* Multiply numbers by 1000
* Divide numbers by 10
* Divide numbers by 100
* Divide numbers by 1000
* Understand and use negative numbers when working in context, such as temperature
* Calculate intervals across zero
* Find common multiples of two numbers
* Find common factors of two numbers
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand and use place value in numbers with up to seven digits
* Multiply and divide whole numbers by 10, 100, 1000
* Multiply and divide numbers with one decimal place by 10, 100, 1000
* Know the meaning of ‘factor’ and ‘multiple’ and ‘prime’

**Bring on the Maths+: Moving on up!**Number and Place Value: #1Solving problems: #3 | Place valueDigitNegative number(Common) multiple(Common) factorDivisiblePrime number, Composite number | Zero is neither positive nor negative.When multiplying and dividing by powers of ten, the decimal point is fixed and it is the digits that move.Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 43 006 619).It is helpful to read ‘4 - -2’ as the difference between 4 and ‘negative 2’ to understand the answer is 6 rather than encouraging misleading rules such as ‘two minuses make a plus’.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.**Every classroom has a set of* [*number classification posters*](http://kangaroomaths.com/free_resources/display/number_classification.pdf)*, a place chart and* a [*negative number washing line*](http://kangaroomaths.com/free_resources/display/number_line.docx) *on the wall*. |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that 109 is a prime number
* Jenny writes 2.54 × 10 = 25.4. Kenny writes 2.54 × 10 = 25.40. who do you agree with? Explain why.
* Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another …
* Convince me a common factor of 12 and 30 is 6.

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | KM: [Maths to Infinity: Directed numbers](http://kangaroomaths.com/free_resources/infinity/directed_numbers.xlsm)KM: [Reading and writing](http://www.kangaroomaths.com/free_resources/teaching/number/read_write.docx)KM: Extend the idea of [Eratosthenes' sieve](http://kangaroomaths.com/free_resources/teaching/number/eratosthenes_sieve.xlsx) to a 12 by 12 gridKM: [Exploring primes activities](http://kangaroomaths.com/free_resources/teaching/number/exploring_primes.docx): Artistic Eratosthenes sieveKM: Use [Powers of ten](http://kangaroomaths.com/free_resources/teaching/number/powers_of_ten.xlsx) to demonstrate connections.NRICH: [Factor-multiple chains](http://nrich.maths.org/public/viewer.php?obj_id=5578)NRICH: [The Moons of Vuvv](http://nrich.maths.org/public/viewer.php?obj_id=1066)NRICH: [Round and round the circle](http://nrich.maths.org/public/viewer.php?obj_id=86)NRICH: [Counting cogs](http://nrich.maths.org/6966)**Learning review**KM: [6M1 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M1_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils confuse factors and multiples.
* Some pupils can confuse the language of large (and small) numbers since the prefix ‘milli- means ‘one thousandth’ (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
* Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement
* Some pupils may think that 1 is a prime number
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| *Checking, approximating and estimating* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Number and Place Value progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberPlaceValue.xlsx) |
| * solve problems which require answers to be rounded to specified degrees of accuracy
* use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
* round any whole number to a required degree of accuracy
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore ways of approximating numbers
* Explore ways of checking answers
 | * Approximate any number by rounding to a specified degree of accuracy; e.g. nearest 1, 10, 100, 1000, decimal place, etc
* Understand estimating as the process of finding a rough value of an answer or calculation
* Use estimation to predict the order of magnitude of the solution to a decimal calculation, including decimals
* Check the order of magnitude of the solution to a calculation, including decimals
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000 or 100 000
* Approximate any number with one or two decimal places by rounding to the nearest whole number
* Approximate any number with two decimal places by rounding to the one decimal place
* Estimate addition (subtraction) calculations with up to four digits
 | Approximate (noun and verb)RoundDecimal placeCheckSolutionAnswerEstimate (noun and verb)Order of magnitudeAccurateAccuracy **Notation** The approximately equal symbol (≈) | This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer. It is useful to estimate calculations involving the multiplication and division of numbers up to four-digits by two-digits to make links with the calculation units.Pupils should use numbers up to 10 000 000 in this unit and decimal numbers up to two decimal places. Pupils should be able to round to other specified degrees of accuracy, but not to a specified number of significant figures, which is introduced in Stage 7.Also see big pictures: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) and [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are taught to visualise rounding through the use a number line* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that 67 rounds to 60 to the nearest 20
* Convince me that 1 579 234 rounds to 2 million to the nearest million
* Jenny writes 1359 ÷ 18 ≈ 7.55. Comment on Jenny’s approximation.
* Lenny writes 2.74 × 13 ≈ 26. Do you agree with Lenny? Explain your answer.

NCETM: [Place Value Reasoning](https://www.ncetm.org.uk/public/files/18416215/1_Progression_Map_Place_Value_Reasoning.pdf) | KM: [Stick on the Maths CALC6: Checking results](http://www.kangaroomaths.com/free_resources/teaching/sotm/level4/4calc6_ewb.doc)KM: [Maths to Infinity Rounding](http://www.kangaroomaths.com/free_resources/infinity/rounding.xlsm)NRICH: [Four Go](http://nrich.maths.org/5633)NCETM: [Activity A(i)](https://www.ncetm.org.uk/resources/42506)NCETM: [Activity G](https://www.ncetm.org.uk/resources/42612)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may truncate instead of round
* When checking the order of magnitude of a division calculation some pupils may apply incorrect reasoning about the effect of increasing the divisor by a factor of 10, thinking that it also makes the solution greater by a factor of 10; e.g. 1400 ÷ 20: 1400 ÷ 2 = 700 so 1400 ÷ 20 = 7000.
* Some pupils may round down at the half way point, rather than round up.
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| *Calculating* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * perform mental calculations, including with mixed operations and large numbers
* solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
* multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
* solve problems involving addition, subtraction and multiplication
* use their knowledge of the order of operations to carry out calculations
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop mental calculation skills
* Extend written methods of multiplication
* Know and use the order of operations
* Solve problems involving addition, subtraction and multiplication

**Bring on the Maths+: Moving on up!**Calculating: #4Fractions, decimals & percentages: #6Solving problems: #2 | * Carry out calculations mentally involving numbers up to 4 digits.
* Multiply a four-digit number by a two-digit number using long multiplication
* Carry out calculations involving a mixture of multiplication and division
* Carry out calculations involving mixture of addition and subtraction
* Carry out calculations involving mixture of multiplication and addition/subtraction
* Carry out calculations involving mixture of division and addition/subtraction
* Solve multi-step problems involving addition, subtraction and/or multiplication
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| Prerequisites | Mathematical language | Pedagogical notes |
| * Recall multiplication facts for multiplication tables up to 12 × 12
* Recall division facts for multiplication tables up to 12 × 12
* Understand the commutativity of multiplication and addition
* Multiply a three-digit number by a two-digit number using long multiplication
* Use column addition and subtraction for numbers with more than four digits

**Bring on the Maths+: Moving on up!**Calculating: #1Solving problems: #1 | AdditionSubtractionSum, TotalDifference, Minus, LessColumn additionColumn subtractionOperationMultiply, MultiplicationTimesProductCommutativeFactorShort multiplicationLong multiplicationEstimate | Pupils understand ‘mentally’ means ‘in your head’ or ‘with jottings’.Note that if not understood fully, the order of operations can give the wrong answer to a calculation; e.g. 6 – 2 x 3.Use a basic and a scientific calculator to work out 2 + 3 × 5. Why are the answers different?Later in this stage there is chance to develop and practice these skills with an emphasis on checking, approximating or estimating the answer.KM: [Progression: Addition and Subtraction](http://kangaroomaths.com/free_resources/hod/bouncebuzz_addition_subtraction_v4.pdf), [Progression: Multiplication and Division](http://kangaroomaths.com/free_resources/hod/bouncebuzz_multiplication_division_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44567), [Subtraction](https://www.ncetm.org.uk/resources/40532), [Multiplication](https://www.ncetm.org.uk/resources/40530), [Multiplicative reasoning](https://www.ncetm.org.uk/resources/43669), [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All classrooms display a* [*times table poster with a twist*](http://kangaroomaths.com/free_resources/display/chinese_tables.docx)*To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition. Teachers say ‘complete these calculations’ instead of ‘complete these sums’**Long multiplication is promoted as the ‘most efficient method’.* *If any acronym is promoted to help remember the order of operations, then BIDMAS is used to strengthen progression as the I stands for indices.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Find missing digits in otherwise completed long multiplication calculations
* Convince me that 2472 × 12 = 29664
* Why have you chosen to add (subtract, multiply)?

NCETM: [Addition and Subtraction Reasoning](https://www.ncetm.org.uk/public/files/18416326/2_Progression_Map_Addition_and_Subtraction_Reasoning.pdf)NCETM: [Multiplication and Division Reasoning](https://www.ncetm.org.uk/public/files/18438909/3_Progression_Map_Multiplication_and_Division_Reasoningv2.pdf) | KM: [Long multiplication template](http://kangaroomaths.com/free_resources/teaching/number/long_multiplication_template.docx)KM: [Maximise, minimise](http://kangaroomaths.com/free_resources/teaching/number/maxmin.docx). Adapt ideas to fit learning intentions.KM: [Maths to Infinity: Complements](http://kangaroomaths.com/free_resources/infinity/complements.xlsm)KM: [Maths to Infinity: Multiplying and dividing](http://kangaroomaths.com/free_resources/infinity/multiply_divide.xlsm)NRICH: [Become Maths detectives](http://nrich.maths.org/6928)NRICH: [Exploring number patterns you make](http://nrich.maths.org/8387)NRICH: [Reach 100](http://nrich.maths.org/public/viewer.php?obj_id=1130)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write statements such as 140 - 190 = 50
* When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
* The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.
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| *Calculating: division* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Calculation progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberCalculation.xlsx) |
| * divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
* divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
* use written division methods in cases where the answer has up to two decimal places
* solve problems involving division
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Develop written methods of short division for numbers up to four-digits divided by a one-digit number
* Deal with remainders when carrying out division
* Solve problems involving the four operations

**Bring on the Maths+: Moving on up!**Calculating: #5 | * Divide a three-digit number by a two-digit number using a formal written method of division with no remainder
* Divide a three-digit number by a two-digit number using a formal written method of division with a remainder
* Divide a four-digit number by a two-digit number using a formal written method of division with no remainder
* Divide a four-digit number by a two-digit number using a formal written method of division with a remainder
* Understand how to write the remainder to a division problem as a whole number remainder or as a fraction
* Understand how to interpret remainder to a division problem appropriately for the context
* Solve problems involving division
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use knowledge of multiplication tables when dividing
* Know how to use short division
 | CommutativeDivide, Division, DivisibleDivisor, Dividend, Quotient, RemainderFactorShort divisionLong divisionRemainderOperationEstimate**Notation**Remainders are often abbreviated to ‘r’ or ‘rem’ | Long division is a method of division that formalises how to find a remainder at each step. Short division is a compact method that uses pupils’ abilities to find this remainder without the need for a formal written method. To help pupils with formal written methods for division it is useful for pupils to know how to find multiples of the divisor by making connections – ie If I know 1x, I know 10x. If I know 10x, I know 5x. If I know 1x, I know 2x. If I know 2x, I know 4x. If I know 4x, I know 8x, etcLater in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.KM: [Progression: Multiplication and Division](http://kangaroomaths.com/free_resources/hod/bouncebuzz_multiplication_division_v4.pdf) and [Calculation overview](http://kangaroomaths.com/free_resources/hod/bouncebuzz_calculation_overview_v4.pdf)NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44568), [Division](https://www.ncetm.org.uk/resources/43589), [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Short division is promoted as the ‘most efficient method’.**When dealing with remainders in division problems, use the notation ‘r’* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Find missing digits in otherwise completed long / short division calculations
* Show me a calculation that is connected to 147 × 26 = 3822. And another, and another …
* Show me a division calculation that has no remainder. Now show me a division by a two-digit number that has no remainder. And now, a four-digit number divided by a two-digit number that has no remainder. And now, with a remainder of 3 …

NCETM: [Multiplication and Division Reasoning](https://www.ncetm.org.uk/public/files/18438909/3_Progression_Map_Multiplication_and_Division_Reasoningv2.pdf) | KM: [Dividing (lots)](http://kangaroomaths.com/free_resources/teaching/number/dividing_lots.docx)KM: [Maths to Infinity: Multiplying and dividing](http://kangaroomaths.com/free_resources/infinity/multiply_divide.xlsm)KM: [Interactive long division](http://kangaroomaths.com/free_resources/teaching/number/long_division.xlsx)KM: [Interactive target boards](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_Interactive_Target_Boards.xlsm)KM: [Maths to Infinity: Multiplication and division foundations](http://kangaroomaths.com/free_resources/teaching/number/KangarooMaths_The%20Foundations_MultDiv.xlsm)NRICH: [Interactive division](http://nrich.maths.org/5400)NRICH [Dicey Operations](http://nrich.maths.org/6606). Game 6.**Learning review**KM: [6M2 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M2_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may write statements such as 12 ÷ 132 = 11
* Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.
* When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

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| --- | --- | --- | --- | --- |
|  | 0 | 10 | 7 | r 5 |
| 8 | 3 | 86 | 61 |  |

3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been ‘moved across’ and therefore everything that follows has been correctly carried out based on an early misunderstanding. |

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| *Visualising and constructing* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * draw 2-D shapes using given dimensions and angles
* recognise, describe and build simple 3-D shapes, including making nets
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Construct 2D shapes
* Investigate 3D shapes
* Explore nets of 3D shapes

**Bring on the Maths+: Moving on up!**Properties of shapes: #1, #2, #3, #4 | * Draw 2-D shapes given angles
* Draw 2-D shapes given dimensions and angles
* Recognise prisms
* Recognise pyramids
* Classify 3-D shapes including cylinders, cones and spheres
* Draw nets of 3-D shapes
* Solve 3-D problems using nets including visualising the edges (vertices) that will meet when folded
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the names of common 2D shapes
* Know the names of common 3D shapes
* Use a protractor to measure and draw angles
 | ProtractorMeasureNearestConstructSketchCube, Cuboid, Cylinder, Pyramid, PrismNetEdge, Face, Vertex (Vertices)Visualise**Notation**Dash notation to represent equal lengths in shapes and geometric diagramsRight angle notation | A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.A cube is a special case of a cuboid, and a cuboid is a special case of a prism.Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a set of* [*3D shape posters*](http://kangaroomaths.com/free_resources/display/solids.pdf) *on the wall**Models of 3D shapes to be used by all students during this unit of work**All pupils to use ‘Polydron’ to explore nets of 3D shapes during this unit of work* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an example of a net of a cube. And another. And another …
* What is wrong with this attempt at a net of a cuboid? How can it be changed?
* How many different ways are there to [complete these nets](http://kangaroomaths.com/free_resources/teaching/geometry/complete_net.pptx)?
* Convince me a cylinder is not a prism.

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: Investigate ‘Platonic Solids’KM: [Visualising 3D shapes](http://kangaroomaths.com/free_resources/teaching/geometry/visualising_3d_shapes.docx)KM: [Tessellating Tess](http://kangaroomaths.com/free_resources/teaching/geometry/tessellating_tess.docx)KM: [Fibonacci’s disappearing squares](http://kangaroomaths.com/free_resources/teaching/geometry/fib_disappearing_squares.docx)KM: [Unravelling dice](http://kangaroomaths.com/free_resources/teaching/geometry/unravelling_dice.docx)NRICH: [Making spirals](http://nrich.maths.org/8294)NRICH: [Cut nets](http://nrich.maths.org/public/viewer.php?obj_id=2315)NRICH: [Making cuboids](http://nrich.maths.org/public/viewer.php?obj_id=90)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle
* Some pupils may measure from the end of a ruler, rather than the start of the measuring scale
* Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation
* When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of ‘corners’ on the net
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| *Investigating properties of shapes* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Properties of Shape progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPropertiesShape.xlsx) |
| * compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
* illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Investigate properties of 2D shapes
* Investigate angles in polygons
* Understand and use the vocabulary of circles

**Bring on the Maths+: Moving on up!**Properties of shapes: #5 | * Classify 2D shapes using given categories; e.g. number of sides, symmetry
* Find unknown angles in a triangle
* Find unknown angles in an isosceles triangle when only one angle is known
* Find unknown angles in a quadrilateral
* Find unknown angles in regular polygons
* Solve problems involving 2-D shapes
* Know the names and relationships of the parts a circle
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the properties of rectangles
* Know the difference between a regular and an irregular polygon
* Add and subtract numbers up to three digits
 | Quadrilateral, Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus, Delta, ArrowheadTriangle, Scalene, Right-angled, Isosceles, EquilateralPolygon, Regular, IrregularPentagon, Hexagon, Octagon, Decagon, DodecagonCircle, Radius, Diameter, Circumference, CentreParallelDiagonalAngle**Notation**Dash notation to represent equal lengths in shapes and geometric diagramsRight angle notation | Ensure that pupils do not use the word ‘diamond’ to describe a kite, or a square that is 45° to the horizontal. ‘Diamond’ is not the mathematical name of any shape. A square is a special case of a rectangle. An oblong is a rectangle that is not a square. A rhombus is a special case of a parallelogram. All polygons up to 20 sides have names, although many have alternatives based on either Latin or Greek.Pupils are expected to know the angle sum of a triangle, angle sum of a quadrilateral and the angle sum of a any polygon. Splitting any polygon into the minimum number of triangles (by drawing all diagonals from one vertex) will allow pupils to find the angle sum of the polygon.This video is (Grade 4 What is a circle?) is very useful to explore 'what makes a circle a circle’.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All teachers refer to a ‘delta’ instead of an ‘arrowhead’**Every classroom has a set of* [*triangle posters*](http://kangaroomaths.com/free_resources/display/triangles.pdf) *and* [*quadrilateral posters*](http://kangaroomaths.com/free_resources/display/quadrilaterals.pdf) *on the wall* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Convince me that a rhombus is a parallelogram
* Jenny writes that ‘Diameter = 2 × Radius’. Kenny writes that ‘Radius = 2 × Diameter’. Who is correct?
* What is the same and what is different: a square and a rectangle?

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: Investigate ‘Tessellations’KM: Explore pi (ratio of the circumference to the diameter)KM: [Shape work](http://kangaroomaths.com/free_resources/teaching/geometry/shape_work.docx): Many of the activities are suitable for this unit.KM: [Dotty activities](http://kangaroomaths.com/free_resources/teaching/geometry/3x3_4x4_5x5_dotty_activities.docx)KM: [Investigating polygons](http://kangaroomaths.com/free_resources/teaching/geometry/investigating_polygons.docx). Tasks one and two.KM: [Special polygons](http://kangaroomaths.com/free_resources/teaching/geometry/special_polygons.docx)NRICH: [Where Are They?](http://nrich.maths.org/public/viewer.php?obj_id=1058)NRICH: [Round a Hexagon](http://nrich.maths.org/8095)NRICH: [Quadrilaterals](http://nrich.maths.org/public/viewer.php?obj_id=962)KM: [6 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/6_point_circles.pdf), [8 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/8_point_circles.pdf) and [12 point circles](http://kangaroomaths.com/free_resources/teaching/geometry/12_point_circles.pdf) can be used to support and extend the above idea**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that a ‘regular’ polygon is a ‘normal’ polygon
* Some pupils may think that all polygons have to be regular
* Some pupils may think that a square is only square if ‘horizontal’, and even that a ‘non-horizontal’ square is called a diamond
* The equal angles of an isosceles triangle are not always the ‘base angles’ as some pupils may think
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| *Algebraic proficiency: using formulae* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * use simple formulae
* convert between miles and kilometres
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Use simple formulae written in words
* Create simple formulae written in words
* Work with formulae written algebraically

**Bring on the Maths+: Moving on up!**Algebra: #1 | * Use a simple one-step formula written in words
* Use a simple two-step formula written in words
* Use simple formula expressed in symbols
* Convert between miles and kilometres
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the order of operations
* Know the fact that area of rectangle = length × width
 | Formula, FormulaeExpressionVariableSubstituteSymbolMileKilometreMetricImperial**Notation**When written algebraically a formula should not include any units. | Pupils have already used the written formula ‘area of rectangle = length × width’. This can be used here to introduce the use of letters to represent variables; ‘A = l × w’. Later in the year pupils will meet other formulae for area and volume and this unit should be used to develop conceptual understanding in readiness for this. Other common examples that could be used include the rough conversion between miles and kilometres, ‘kilometres = miles × 1.6’.NCETM: [Algebra](https://www.ncetm.org.uk/resources/43649)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches** |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Look at this formula. Write down a fact that it tells you. And another. And another …
* Jenny and Kenny are using the formula ‘Cost in pounds = 40 + 20 × number of hours’ to work out the cost for three hours. Jenny writes down £180. Kenny writes down £100. Who do you agree with? Why?
* Always / Sometimes / Never: The formula T = 4n + 6 results in an odd number.

NCETM: [Algebra](https://www.ncetm.org.uk/public/files/18416474/6_Progression_Map_Algebra_Reasoning.pdf) Reasoning | KM: [Fascinating food](http://kangaroomaths.com/free_resources/teaching/algebra/fascinating_food.docx)NCETM: [Year 6 Algebra](https://www.ncetm.org.uk/resources/42893). Activities A and D.**Learning review**KM: [6M3 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M3_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may apply the order of operations incorrectly when working with two step formulae
* Units must be consistent when using formulae. For example, a mobile phone plan might charge £15 per month plus 5p for every text. The formula ‘Monthly cost = 15 + 5 × number of texts is wrong because amounts in both pounds and pence are involved. Monthly cost (in pence) = 1500 + 5 × number of texts is one correct way of writing the formula.
* It is not advisable to abbreviate the formula ‘kilometres = miles × 1.6’ using letters. ‘m’ is the normal abbreviation for metres and ‘k’ can represent £1000. If ‘km’ is used it could even be interpreted as ‘k × m’.
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| *Exploring fractions, decimals and percentages* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * use common factors to simplify fractions; use common multiples to express fractions in the same denomination
* compare and order fractions, including fractions > 1
* associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]
* recall and use equivalences between simple fractions, decimals and percentages, including in different contexts
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore the equivalence between fractions
* Use the equivalence between fractions
* Explore the equivalence between fractions, decimals and percentages

**Bring on the Maths+: Moving on up!**Fractions, decimals & percentages: #1, #2 | * Use common factors to simplify fractions
* Use common multiples to find equivalent fractions
* Compare and order fractions
* Compare and order fractions, including fractions > 1
* Understand a fraction is associated with division
* Work out the decimal equivalents of fifths, eighths and tenths
* Know simple fractions, decimals and percentages equivalences (e.g. 10%, 20%, 25%, 50%, 75%, 100%)
* Find equivalencies between fractions, decimals and percentages
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Understand the concept of a fraction as a proportion
* Understand the concept of equivalent fractions
* Understand the concept of fractions, decimals and percentages being equivalent
* Know standard fraction / decimal equivalences (e.g. ½ = 0.5, ¼ = 0.25, 1/10 = 0.1)
* Know that a percentage means ‘out of 100’
 | FractionImproper fraction, Proper fraction, Vulgar fraction, Top-heavy fractionPercentageDecimalProportionSimplifyEquivalentLowest terms**Notation**Diagonal fraction bar / horizontal fraction bar | Use language carefully to avoid later confusion: when simplifying fractions, the language ‘divide by 4’ should not be used in place of ‘divide the top and bottom by 4’. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4.NRICH: [Teaching fractions with understanding](http://nrich.maths.org/2550)NCETM: [Teaching fractions](https://www.ncetm.org.uk/resources/44490)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are made aware that ‘per cent’ is derived from Latin and means ‘out of one hundred’**Teachers use the horizontal fraction bar notation at all times* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me another fraction that is equivalent to this one. And another. And another …
* Convince me that 3/8 = 0.375
* If you know that 1/10 = 0.1 = 10%, what else can you work out?
* Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, ‘if I cancel out the sixes then 16/64 = 1/4.’. Do you agree with Jenny? Why?

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf) | KM: [FDP conversion](http://kangaroomaths.com/free_resources/teaching/number/fdp_conversion.docx)KM: [Carpets](http://kangaroomaths.com/free_resources/teaching/number/fdp_carpets.docx)KM: [Fraction and decimal tables](http://kangaroomaths.com/free_resources/teaching/number/fd_tables.docx)NRICH: [Matching fractions](http://nrich.maths.org/8283)NRICH: [Fractions made faster](http://nrich.maths.org/4561)**Learning review**KM: [6M6 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M6_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.
* Pupils may not make the connection that a percentage is a different way of describing a proportion
* Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)
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| *Proportional reasoning* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Ratio and Proportion progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_RatioProportion.xlsx) |
| * solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
* solve problems involving similar shapes where the scale factor is known or can be found
* solve problems involving unequal sharing and grouping using knowledge of fractions and multiples
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve problems involving scaling
* Explore enlargement
* Solve problems involving sharing and grouping

**Bring on the Maths+: Moving on up!**Ratio and proportion: #1**Y7 Bring on the Maths** Problem Solving: #1, #2, #3 | * Solve simple problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts; e.g. find the value of the parts, given the whole)
* Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts; e.g. find the value of the whole and parts, given one part)
* Use a scale factor to solve problems involving similar shapes
* Find the scale factor of similar shapes
* Solve problems involving unequal sharing or grouping problems using fractions
* Solve problems involving unequal sharing or grouping problems using multiples
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Recall multiplication facts for multiplication tables up to 12 × 12
* Recall division facts for multiplication tables up to 12 × 12
* Find fractions of an amount
* Find multiples of a given number
 | ProportionQuantityIntegerSimilar (shapes)EnlargementScale factorGroupShareMultiples | Any work on enlargement should only include enlargements using a scale factor. The concept of a centre of enlargement is a future development. NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44568)NCETM: [Multiplicative reasoning](https://www.ncetm.org.uk/resources/43669)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils are explicitly taught to use the bar model as a way to represent a problem involving proportional reasoning* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another …
* Convince me that the second shape is an enlargement of the first shape
* Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Jenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why.

NCETM: [Ratio and Proportion Reasoning](https://www.ncetm.org.uk/public/files/18416446/5_Progression_Map_Ratio_and_Proportion_Reasoning.pdf) | NRICH: [Orange Drink](http://nrich.maths.org/public/viewer.php?obj_id=2420) NRICH: [Pumpkin Pie Problem](http://nrich.maths.org/public/viewer.php?obj_id=1026)NRICH: [Jumping](http://nrich.maths.org/7407)NCETM: [Activity Set A](https://www.ncetm.org.uk/resources/42670)NCETM: [Activity Set B](https://www.ncetm.org.uk/resources/42670)NCETM: [Activity Set C](https://www.ncetm.org.uk/resources/42670)NCETM: [Activity Set D](https://www.ncetm.org.uk/resources/42670)**Learning review**KM: [6M5 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M5_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
* When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.
* When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, ‘enlarging’ a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.
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| *Pattern sniffing* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * generate and describe linear number sequences
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore number sequences

**Bring on the Maths+: Moving on up!**Number and Place Value: #4Number and Place Value: #5 | * Recognise and describe a linear sequence
* Find the next terms in a linear sequence
* Find a missing term in a linear sequence
* Generate a linear sequence from its description
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Count forwards and backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)
* Count forwards and backwards through zero
 | PatternSequenceLinearTermAscendingDescending | Pupils have counted forwards and backwards in previous years and units, but this is the first time that the concept of sequences appears specifically.The language ‘term-to-term rule’ should not be introduced until Stage 7.NCETM: [Algebra](https://www.ncetm.org.uk/resources/43649)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers and pupils refer to numbers less than zero as ‘negative’ numbers and not ‘minus’ numbers* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a (ascending/descending) linear sequence. And another. And another.
* Kenny thinks that 2, 4, 8, 16, … is a linear example. Do you agree? Explain your answer.
* Create a linear sequence with a 3rd term of ‘8’.
* Show me a linear sequence where the rule to get from one term to the next is ‘add 3’. And another. And another.

NCETM: [Algebra](https://www.ncetm.org.uk/public/files/18416474/6_Progression_Map_Algebra_Reasoning.pdf) Reasoning | KM: [Maths to Infinity: Sequences](http://kangaroomaths.com/free_resources/infinity/sequences.xlsm)NRICH: [Times Tables Shifts](http://nrich.maths.org/6863)NRICH:[Domino Sets](https://nrich.maths.org/9965)NCETM: [Activity B: Sticky Triangles](https://www.ncetm.org.uk/resources/42893)NCETM: [Activity D: Generating Sequences](https://www.ncetm.org.uk/resources/42893)**Learning review**KM: [6M4 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M4_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think linear sequences are only ascending.
* Some pupils may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, e.g. 2, 4, 8, 16, …
* Some pupils may not appreciate that both a starting number and a rule to find the next term are required in order to describe a sequence in full.
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| *Measuring space* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve problems involving measurement

**Bring on the Maths+: Moving on up!**Measures: #3 | * Convert between non-adjacent metric units length and mass from the smaller unit to the larger unit; e.g. centimetres to kilometres
* Convert between non-adjacent metric units length and mass from the larger unit to the smaller unit; e.g. kilometres and centimetres
* Convert between non-adjacent time units; e.g. hours to seconds
* Solve problems involving converting between measures
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Convert between adjacent metric units of length, mass and capacity
* Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml
* Use decimal notation to two decimal places when converting between metric unit
 | Length, distanceMass, weightVolumeCapacityMetre, centimetre, millimetreTonne, kilogram, gram, milligramLitre, millilitreHour, minute, secondInch, foot, yardPound, ouncePint, gallon**Notation**Abbreviations of units in the metric system: m, cm, mm, kg, g, l, mlAbbreviations of units in the Imperial system: lb, oz | Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.The prefix ‘centi-‘ means one hundredth, and the prefix ‘milli-‘ means one thousandth. These words are of Latin origin.The prefix ‘kilo-‘ means one thousand. This is Greek in origin.Conversion of volumes will be covered in the ‘calculating space’ unit.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a metric (imperial) unit of measure. And another. And another.
* Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer.
* Convince me that 4.25kg does not equal 425g.

NCETM: [Measurement Reasoning](https://www.ncetm.org.uk/public/files/18436766/7_Progression_Map_Measurement_Reasoning.pdf) | KM: [Weighing up the options](http://kangaroomaths.com/free_resources/teaching/geometry/weigh_up_the_options.docx)NRICH: [Place Your Orders](http://nrich.maths.org/7500)NRICH: [Thousands and Millions](http://nrich.maths.org/6046)NCETM: [Activity E : A little bit of history - Marco Polo](https://www.ncetm.org.uk/resources/42805)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
* Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’ clock
* Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.
* Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an ‘extra unit’ within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.
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| *Investigating angles* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles
 |
| [Return to overview](#Overview) |
| Possible themes  | Possible key learning points |
| * Develop knowledge of angles
* Apply angle facts to deduce unknown angles
 | * Find missing angles where they meet at a point
* Find missing angles where they meet on a straight line
* Find missing angles where they are vertically opposite
* Use known facts to find missing angles
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know that angles are measured in degrees
* Know that angles in a full turn total 360°, and angle in half a turn must total 180°
* Estimate the size of angles
 | AngleDegreesRight angleAcute angleObtuse angleReflex angleProtractorVertically opposite**Notation**Right angle notationArc notation for all other anglesThe degree symbol (°) | The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors.The SI unit for measuring angles in the radian (2π radians in a full turn). Napoleon experimented with the decimal degree, or grad (400 grads in a full turn)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***All pupils know how to to use a 180° and a 360° protractor.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show a pair of possible values for a and b.

aabAnd another. And another.  * Convince me that the sum of angles on a straight line is 180°.
* Show a possible set of values for a, b, c and d.

And another. And another.  aabdc* Convince me that the sum of angles around a point is 360°.
* Convince me that (vertically) opposite angles are equal.
* Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer.

NCETM: [Geometry - Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Maths to Infinity: Lines and Angles](http://www.kangaroomaths.com/free_resources/infinity/angles_lines.xlsm)**Learning review**KM: [6M10 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M10_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that these angles are not equal as they are not ‘vertical’.

xx* Some pupils may think that angles that are ‘roughly’ opposite are always equal, e.g. a = c

aabdc |

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| *Calculating fractions, decimals and percentages* | *12 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Fractions, decimals and percentages progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_NumberFDP.xlsx) |
| * add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
* multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8]
* divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6]
* multiply one-digit numbers with up to two decimal places by whole numbers
* solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Calculate with fractions
* Calculate with decimals
* Calculate with percentages

**Bring on the Maths+: Moving on up!**Fractions, decimals & percentages: #3, #4, #7Ratio and proportion: #2 | * Add fractions with different denominators
* Add a mixed number and a fraction, including with different denominators
* Add mixed numbers, including with different denominators
* Subtract fractions with different denominators
* Subtract a mixed number and a fraction, including with different denominators
* Subtract mixed numbers, including with different denominators
* Multiply a proper fraction by a proper fraction
* Divide a proper fraction by a whole number
* Multiply U.t by U
* Multiply U.th by U
* Calculate percentages of a quantity
* Solve problems involving the use of percentages to make comparisons
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Convert between mixed numbers and improper fractions
* Find equivalent fractions
* Add and subtract fractions when one denominator is a multiple of the other
* Multiply a proper fraction by a whole number
* Use the formal written method of short multiplication
* Know the effect of multiplying and dividing by 10 and 100
* Know percentage equivalents of 1/2, 1/4, 3/4, 1/5, 2/5, 4/5

**Bring on the Maths+: Moving on up!**Fractions, decimals & percentages: #5 | Mixed numberEquivalent fractionSimplify, cancelLowest termsProper fractionImproper fraction, top-heavy fractionVulgar fractionNumerator, denominatorPercent, percentage**Notation**Mixed number notationHorizontal / diagonal bar for fractions | Use of a fraction wall to visualise multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as $\frac{1}{4}×\frac{1}{2}$ as $\frac{1}{4}$ multiplied by $\frac{1}{2}$ and therefore, $\frac{1}{2} $of $\frac{1}{4}= \frac{1}{8}$; $\frac{4}{10}÷2$ as $\frac{4}{10} $divided by 2 and therefore $\frac{2}{10}$.NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44568)NCETM: [Teaching fractions](https://www.ncetm.org.uk/resources/44490)NCETM: [Fractions videos](https://www.ncetm.org.uk/resources/43609)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy.**When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy.**Pupils are encouraged to find and use 10% of an amount.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me an ‘easy’ (‘difficult’) pair of fractions to add (subtract). And another. And another.
* Kenny thinks that $\frac{7}{10}- \frac{2}{7}= \frac{5}{3}$ = 1$\frac{2}{3}$. Do you agree with Kenny?
* Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.
* Benny thinks that $\frac{4}{10}÷2=\frac{4}{5} .$ Do you agree with Kenny? Explain.
* Lenny says *‘20% of £60 is £3 because 60* $÷$*20 = 3’*. Do you agree?

NCETM: [Fractions Reasoning](https://www.ncetm.org.uk/public/files/18416412/4_Progression_Map_Fractions_Reasoning_.pdf)NCETM: [Ratio and Proportion Reasoning](https://www.ncetm.org.uk/public/files/18416446/5_Progression_Map_Ratio_and_Proportion_Reasoning.pdf) | NRICH: [Fractions Jigsaw](http://nrich.maths.org/public/viewer.php?obj_id=5467&part=index&refpage=monthindex.php)NRICH: [Peaches Today, Peaches Tomorrow](http://nrich.maths.org/2312/index)NRICH: [Andy’s Marbles](http://nrich.maths.org/public/viewer.php?obj_id=2421)NRICH: [Would you Rather?](http://nrich.maths.org/public/viewer.php?obj_id=1118)**Learning review**KM: [6M7 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M7_BAM.pdf), [6M8 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M8_BAM.pdf), [6M9 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M9_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. 3$\frac{1}{3}$ - 2$\frac{1}{2}=1\frac{-1}{6}$
* Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
* Some pupils may think that as you divide by 10 to find 10%, you divide by 15 to find 15%, divide by 20 to find 20%, divide by 100 to find 100%, etc.
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| *Solving equations and inequalities* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Algebra progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Algebra.xlsx) |
| * enumerate possibilities of combinations of two variables
* express missing number problems algebraically
* find pairs of numbers that satisfy an equation with two unknowns
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Solve missing number problems
* Understand and use algebra

**Bring on the Maths+: Moving on up!**Algebra: #2 | * Find all combinations of two variables that solve a missing number problem with two unknowns
* Find pairs of numbers that satisfy an equation with two unknowns e.g. a + b = 15
* Know the basic rules of algebraic notation
* Express and solve missing number problems algebraically
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use symbols to represent variables in a formula
 | Algebra, algebraic, algebraicallySymbolExpressionVariableSubstituteEquationUnknownEnumerate**Notation**The lower case and upper case of a letter should not be used interchangeably when worked with algebraJuxtaposition is used in place of ‘×’. 2*a* is used rather than *a*2.Division is written as a fraction | The word ‘algebra’ comes from the title of a book by the Persian mathematician, al-Khwārizmī, who lived in modern-day Baghdad about 1200 years ago. Al-kitāb al-mukhtaṣar fī ḥisāb al-ğabr wa’l-muqābala was a book that promoted the idea of solving equations by a method of balancing.Avoid fruit salad algebra (see possible misconceptions).NCETM: [The Bar Model](https://www.ncetm.org.uk/resources/44567)NCETM: [Algebra](https://www.ncetm.org.uk/resources/43649)NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Use ‘think of a number’ problems to introduce the idea of ‘n’ standing for any number (a variable). For example:** *Think of a number, double it, add 8, halve it and finally subtract your starting number.*

*The answer is always 4. n 🡪 2n 🡪 2n + 8 🡪 n + 4 🡪 4.**Encourage students to try again with different numbers, including large, small, negative, fractional or decimal. Also try varying the instructions to end up with a different number every time.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * *a* + *b* = 15. Show me a pair of values for *a* and *b*. And another. And another.
* *p* + *q* = 7. Show me a pair of values for p and q that no one else will think of. And another. And another.
* Kenny thinks that ‘*b*2 ‘ is the same as ‘2*b*’ because when *b* = 2, *b*2 = 4 and 2*b* = 4. Do you agree with Kenny? Explain your answer.
* Jenny thinks that 7 + 2*a* = 9*a*. Do you agree with Jenny? Explain your answer.

NCETM: [Algebra](https://www.ncetm.org.uk/public/files/18416474/6_Progression_Map_Algebra_Reasoning.pdf) Reasoning | NRICH: [Plenty of Pens](https://nrich.maths.org/1117)NRICH: [Your Number Is…](http://nrich.maths.org/2289)NRICH: [Number Pyramids](http://nrich.maths.org/2281)NCETM: [Activity A: Racetrack and Design a board game](https://www.ncetm.org.uk/resources/42893)NCETM: [Activity E: Matchbox Algebra](https://www.ncetm.org.uk/resources/42893)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think that variables have a set value, such as *a* = 1, *b* = 2, *c*  = 3, *d* = 4, etc. (especially if they have done lots of poorly designed treasure hunts/codes) – this will lead to problems such as thinking ‘*b*2 ‘ is the same as ‘2*b*’ because when *b* = 2, *b*2 = 4 and 2*b* = 4.
* Using the idea of ‘apples’ and ‘bananas’ to explain *a* + *b* = 14 can lead to misconceptions about the use of letters as variables.
* Some students may think that the variables have to be positive integers (whole numbers)
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| *Calculating space* | *8 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Measurement and mensuration progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryMeasurementMensuration.xlsx) |
| * recognise that shapes with the same areas can have different perimeters and vice versa
* calculate the area of parallelograms and triangles
* calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm³) and cubic metres (m³), and extending to other units [for example, mm³ and km³]
* recognise when it is possible to use formulae for area and volume of shape
* solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Explore area
* Investigate volume
* Solve problems involving area and volume

**Bring on the Maths+: Moving on up!**Measures: #6 | * Recognise that shapes with the same areas can have different perimeters and vice versa
* Calculate the area of a parallelogram
* Calculate the area of a triangle
* Estimate the volume of cubes and cuboids
* Calculate the volume of cuboid, including cubes
* Recognise when it is possible to use formulae to calculate area and volume
* Convert between metric units of area in simple cases
* Convert between metric units of volume in simple cases
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Know the meaning of perimeter (area, volume, capacity)
* Know that the area of a rectangle is given by the formula area = length × width
* Know that area can be measured using square centimetres or square metres, and the abbreviations cm2 and m2
* Know that volume is measured in cubes

**Bring on the Maths+: Moving on up!**Measures: #4, #5 | Perimeter, area, volume, capacitySquare, rectangle, parallelogram, triangleComposite rectilinearPolygonCube, cuboidMillimetre, Centimetre, Metre, KilometreSquare millimetre, square centimetre, square metre, square kilometreCubic centimetre, centimetre cubeFormula, formulaeConvertLength, breadth, depth, height, width**Notation**Abbreviations of units in the metric system: km, m, cm, mm, mm2, cm2, m2, km2, mm3, cm3, km3 | Pupils are expected to know that the area of a parallelogram is given by the formula area = base × height; the area of a triangle is given by the formula area = ½ × base × height = base × height ÷ 2 = $\frac{bh}{2}$; the volume of a cuboid is given by the formula volume = length × width × height.In this unit, ‘volumes of shapes’ refers only to cubes and cuboids.Ensure that pupils make connections with the area of a rectangle work in Stage 5, in particular the importance of the perpendicular height.Note that there are several different ways of stating the area of a triangle and this can cause confusionNCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pupils derive practically the formulae for area of parallelogram and triangle by dissecting rectangles**Pupils derive the formula for the area of a parallelogram first. They then use this to help derive the formula for the area of an obtuse-angled triangle.**Every classroom has a set of* [*area posters*](http://kangaroomaths.com/free_resources/display/areas.pdf) *on the wall**Pupils use the area of a triangle as given by the formula area =* $\frac{bh}{2}$ |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * ‘Show me’ an example of when you would measure volume using km3
* Convince me that the area of a parallelogram is found using base × height
* (Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Kenny thinks that the area is 40 cm2, Lenny thinks it is 20 cm2, Jenny thinks it is 240 cm2 and Benny thinks it is 24 cm2. Who do you agree with? Explain why.

NCETM: [Geometry -Properties of Shapes Reasoning](https://www.ncetm.org.uk/public/files/18438967/8_Progression_Map_Geometry_properties_of_shapes_Reasoningv2.pdf) | KM: [Fibonacci’s disappearing squares](http://kangaroomaths.com/free_resources/teaching/geometry/fib_disappearing_squares.docx)KM: [Dissections deductions](http://kangaroomaths.com/free_resources/teaching/geometry/dissection_deductions.docx)KM: [Stick on the Maths SSM9: Area and volume](http://www.kangaroomaths.com/free_resources/teaching/sotm/level6/6ssm9_ewb.doc)KM: [Maths to Infinity Area and Volume](http://www.kangaroomaths.com/free_resources/infinity/area_volume.xlsm)NCETM**:** [Activity C: Through the window](https://www.ncetm.org.uk/resources/42805)**Learning review**KM: [6M11 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M11_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may use the sloping height when finding the areas of parallelograms and triangles
* Some pupils may think that the area of a triangle is found using area = base × height
* Some pupils may think that you multiply all the numbers to find the area of a shape
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| *Mathematical movement* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Position and direction progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_GeometryPositionDirection.xlsx) |
| * describe positions on the full coordinate grid (all four quadrants)
* draw and translate simple shapes on the coordinate plane, and reflect them in the axes
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use Cartesian coordinates
* Use transformations to move shapes
 | * Use coordinates to describe the position of a point in all four quadrants
* Use coordinates to plot the position of a point in any of the four quadrants
* Draw and translate simple shapes
* Carry out a reflection using one of the axes as a mirror line
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Use coordinates in the first quadrant
* Identify a translation
* Carry out a translation in the first quadrant
* Identify a reflection
* Carry out a reflection in the first quadrant using mirror lines parallel to the axes
* Know the meaning of ‘congruent’, ‘congruence’, ‘object’, ‘image’

**Bring on the Maths+: Moving on up!**Properties of shapes: #2Position and direction: #1 | 2-DGridAxis, axes, x-axis, y-axisOriginQuadrant(Cartesian) coordinatesPointTranslationReflectionTransformationObject, ImageCongruent, congruence**Notation**Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) | The main focus of this unit is to develop understanding of coordinates in all four quadrantsNote that pupils are not yet expected to use an algebraic description of a mirror line (such as x = 3).The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.Other coordinate systems include grid references, polar coordinates and spherical coordinates.There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is ‘transformations’.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Teachers do not use the phrase ‘along the corridor and up the stairs’ as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is ‘x is a cross, wise up!’**Teachers use the language ‘negative number’, and not ‘minus number’, to avoid future confusion with calculations.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * (Given a grid with the point (-3, 4) indicated) Benny describes this point as (-3, 4). Jenny describes the point as (4, -3). Who do you agree with? Why?
* Two vertices of a rectangle are (-1, 2) and (4, -2). What could the other two vertices be? How many solutions can you find?
* Convince me that (-2, 3) is in the second quadrant)

NCETM: [Geometry: Position Direction and Movement Reasoning](https://www.ncetm.org.uk/public/files/18436990/9_Progression_Map_Geometry_position_direction_and_movement_Reasoning.pdf) | KM: [Stick on the Maths ALG2: Coordinates in four quadrants](http://www.kangaroomaths.com/free_resources/teaching/sotm/level5/5alg2_ewb.doc)NRICH: [Cops and Robbers](http://nrich.maths.org/public/viewer.php?obj_id=6288)NRICH: [Eight Hidden Squares](http://nrich.maths.org/public/viewer.php?obj_id=6280)NRICH: [Coordinate Tan](http://nrich.maths.org/public/viewer.php?obj_id=1109)NRICH: [Transformation Tease](http://nrich.maths.org/1111)NCETM: [Activity B - Battleships](https://www.ncetm.org.uk/resources/42950)**Learning review**KM: [6M12 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M12_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
* When reflecting a triangle some students may draw a translation
* When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line.
* Some pupils will confuse the order of x-coordinates and y-coordinates
* When constructing axes, some pupils may not realise the importance of equal divisions on the axes
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| *Presentation of data* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * interpret and construct pie charts and line graphs and use these to solve problems
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Construct and interpret pie charts
* Solve problems involving graphs and charts

**Bring on the Maths+: Moving on up!**Statistics: #2, #3 | * Interpret pie charts
* Construct a pie chart by measuring angles
* Interpret line graphs
* Construct line graphs
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Measure and construct angles using a protractor
* Interpret and construct a simple line graph
 | DataScaleAxis, axesGraphFrequencyTime graph, Time seriesLine graphPie chartSectorAngleProtractorDegreesMaximum, minimum | In Stage 6, when constructing pie charts the total of the frequencies is always a factor of 360 More complex cases are included in later stages. It is essential that pupils understand pie charts are used to show proportions and not discrete data.William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. He also introduced the pie chart in 1801.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency, and not using percentages.**The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another.
* Always / Sometimes / Never: Pie charts are constructed in a clockwise direction
* Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency
* Kenny says ‘If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.’ Do you agree with Kenny? Explain your answer.

NCETM: [Statistics Reasoning](https://www.ncetm.org.uk/public/files/18437062/10_Progression_Map_Statistics_Reasoning.pdf) | KM: [Stick on the Maths HD6: Graphs and diagrams](http://www.kangaroomaths.com/free_resources/teaching/sotm/level5/5hd6_ewb.doc)NRICH: [Match the Matches](http://nrich.maths.org/public/viewer.php?obj_id=4937)NRICH: [Graphing Number Patterns](http://nrich.maths.org/1174)NCETM: [A little bit of history (Britain since 1945)](https://www.ncetm.org.uk/resources/38193)**Learning review**NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * Some pupils may think the larger the size of the pie chart, the greater the total frequency
* Some pupils may think if two pie charts have the same section then the amount of data the section represents is the same in each pie chart.’
* Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°
* Some pupils may think that a line graph is appropriate for discrete data
* Some pupils may think that each square on the grid used represents one unit
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| *Measuring data* | *4 lessons* |
| **Key concepts (National Curriculum statements)** | **The Big Picture**: [Statistics progression map](http://kangaroomaths.com/free_resources/planning/KM_MathematicsProgression_Statistics.xlsx) |
| * calculate and interpret the mean as an average
 |
| [Return to overview](#Overview) |
| Possible themes | Possible key learning points |
| * Understand and use the mean

**Bring on the Maths+: Moving on up!**Statistics: #4 | * Understand the meaning of ‘average’ as a typicality (or location)
* Calculate the mean of a set of discrete data
* Interpret the mean of a set of discrete data
* Use the mean to find a missing number in a set of data
 |
| Prerequisites | Mathematical language | Pedagogical notes |
| * Approximate a number by rounding to a given number of decimal places
 | AverageMeanMeasureDataStatisticStatisticsApproximateRound | The word ‘average’ is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the ‘arithmetic mean’). Other types of average, including the mode and the median, are introduced in later stages.NCETM: [Glossary](https://www.ncetm.org.uk/public/files/17308038/National%2BCurriculum%2BGlossary.pdf)**Common approaches***Always use brackets when writing out the calculation for a mean, e.g. (2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5* |
| Reasoning opportunities and probing questions | Suggested activities | Possible misconceptions |
| * Always / Sometimes / Never: The mean is a whole number.
* Kenny is working out the mean of 2, 3, 4 and 5. He calculates 2 + 3 + 4 + 5 ÷ 4 = 10.25. Do you agree with Kenny? Explain your answer.
* The average number of children per family (Married Couples, 2012) is 1.8. Convince me that this statement makes sense.

NCETM: [Statistics Reasoning](https://www.ncetm.org.uk/public/files/18437062/10_Progression_Map_Statistics_Reasoning.pdf) | KM: [Maths to Infinity: Averages, Charts and Tables](http://www.kangaroomaths.com/free_resources/infinity/averages_charts_tables.xlsm)NRICH: [Birdwatch](http://nrich.maths.org/7553)NRICH: [Probably …](http://nrich.maths.org/7245)NRICH: [Same or Different?](http://nrich.maths.org/public/viewer.php?obj_id=1176)NCETM: [A little bit of history (Britain since 1945)](https://www.ncetm.org.uk/resources/38193)**Learning review**KM: [6M13 BAM Task](http://kangaroomaths.com/free_resources/assessment/BAM/6M13_BAM.pdf)NCETM: [NC Assessment Materials (Teaching and Assessing Mastery)](https://www.ncetm.org.uk/resources/46689) | * If using a calculator some pupils may not use the ‘=’ symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as 2 + 3 + 4 + 5 ÷ 4 = 10.25.
* Some pupils may think the average is always the middle number
* Some pupils may think that the mean must be a whole number
* Some pupils may not realise that the mean must lie within the range of the data set.
 |